OU-HET 714/2011

A Simple Method of Calculating Effective Operators

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Abstract

It is important to obtain effective operators by integrating out high energy degrees of freedom in physics. We suggest a general method of calculating accurate irrelevant operators in a scattering process without use of equation of motions. By using this method, for example, we will represent a complete set of dimension six operators in QCD, which are induced from physics beyond the standard model, supersymmetry and universal extra dimension. We will also show an example of effective anomalous 4-Fermi interactions induced from a little Higgs model.

1 Introduction

It is important to obtain effective operators by integrating out high energy degrees of freedom in physics. In a quantum field theory, we can obtain effective Lagrangian by integrating out high energy momentum and heavy particles. And their effects are introduced into irrelevant operators in a low energy effective theory. Therefore, it is important to obtain effective irrelevant operators accurately for a search of new physics. As for high energy physics, search for physics beyond the standard model (SM) is one of the main subjects in Large Hadron Collider (LHC). For both theoretical and numerical analyses to search new physics, we stress again that it is important to obtain accurate irrelevant operators because they include the hints of new physics.

There have been some works of listing higher dimensional operators in the field content of the SM (or also including right-handed neutrinos.) Allowed irrelevant operators in an effective theory should be determined by symmetries existed in the theory. Let us focus on dimension six operators mainly in this paper, and possible dimension six operators within the SM field content were listed in Ref.[1]. Meanwhile, this set was not irreducible, and Refs.[2, 3] have obtained a complete set systematically. However, Ref.[4] insisted that 80 numbers of complete set can be diminishable to 59 by using equation of motions (EOMs). Which set should we use in a calculation of a scattering process? Also, when we obtain effective operator, how to use it for a calculation of a cross section and how to use EOMs in it? Beside a difficulty of estimating correct symmetric factors, there are some above mistakable points when we use irrelevant operators.

In this paper, we show a general method of calculating accurate irrelevant effective operators in a scattering process without use of EOMs. By using this method, we will represent dimension six operators induced from supersymmetry (SUSY)[5], universal extra dimension (UED)[6], and little Higgs model (LH)[7]. We will show coefficients of dimension six operators in QCD by integrating out sparticles (KK particles) in SUSY (UED). We will also show coefficients of 4-Fermi operators originating from anomalous interactions in LH. They are promising candidates of beyond the SM, and our analyses will shed lights on the search of beyond the SM. Our method can apply to other quantum field theories in any dimensions, so that we believe this technique is very useful in a lot of researches in physics.

2 Method of obtaining effective operators

Let us show a general method of obtaining accurate irrelevant operators which are useful for calculating a scattering processes in an effective theory. The effects of beyond the SM must contain in higher order irrelevant operators in general. Thus, when we integrate out new particles and high momentum (physics) above an energy scale of Λ, the effective theory $\mathcal{L}_{\rm eff}$ can be expanded as a power of Λ^{-1} as,

$$\mathcal{L}_{\rm eff} = \mathcal{L}_0 + \frac{1}{\Lambda}\mathcal{L}_1 + \frac{1}{\Lambda^2}\mathcal{L}_2 + \cdots \, , \tag{2.1}$$

where $\mathcal{L}_0$ is the SM Lagrangian, and $\mathcal{L}_1$ ($\mathcal{L}_2$) represents dimension five (six) operators. As for dimension five operator, there is only one operator written within the SM field contents, which induces Majorana neutrino masses. Thus, let us mainly focus on non-trivial next lowest operator, i.e., dimension six operators

$$\mathcal{L}_2 = \sum_i c_i \mathcal{O}_i^{(6)}, \tag{2.2}$$

where c_i is a coefficient, and i is the index of all possible dimension six operators allowed by the SM gauge symmetry. How can we calculate dimension six operators in the effective Lagrangian by integrating out high energy degrees of freedom? One correct answer is to take a path integral of the full theory as

$$Z = \int \mathcal{D}\phi_{SM}\mathcal{D}\phi_h e^{iS[\phi_{SM},\phi_h]}, \tag{2.3}$$

where ϕ_{SM} and ϕ_h represent the SM fields and heavy fields, respectively. By integrating out ϕ_h as

$$e^{iS_{\text{eff}}[\phi_{SM}]} = \int \mathcal{D}\phi_h e^{iS[\phi_{SM},\phi_h]}, \tag{2.4}$$

we can obtain an effective action,

$$S_{\text{eff}}[\phi_{SM}] = S_{SM} + S_1[\phi_{SM}] + S_2[\phi_{SM}] + \cdots , \tag{2.5}$$

where $S_1 = (1/\Lambda)\int \mathcal{L}_1$ and $S_2 = (1/\Lambda^2)\int \mathcal{L}_2$. Coefficients of dimension six operators have been basically calculated in $S_2[\phi_{SM}]$, by which a S-matrix element in this effective theory could be estimated. Before showing a concrete calculation method, we consider a role of EOMs when we calculate irrelevant operators.

Within the SM field content, one complete set of $\mathcal{O}^{(6)}$ is explicitly listed in Refs[2, 3], where coefficients are not determined until we fix a fundamental theory existing behind the SM. However, Ref.[4] insisted the 80 numbers of dimension six operators of the complete set can be diminishable to 59 numbers by using EOMs. Which set should we use in a calculation of a scattering process? To answer this question, we must know correctly whether we can use EOMs in a calculation of irrelevant operators or not. Let us overview arguments of Ref.[4] at first, where they use the fact that dimension six operators are related among each others through classical EOMs. For example, let us consider quark-quark-gluon interaction in dimension six operators [1] as

$$\mathcal{O}_{qG} = (\overline{q}\gamma^\mu T^a q)(iD^\nu G_{\mu\nu})^a, \tag{2.6}$$

where $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$, $(D^\nu G_{\mu\nu})^a = \partial^\nu G^a_{\mu\nu} + g_s f^{abc} G^{b\nu} G^c_{\mu\nu}$, and EOM is given by

$$(D^\nu G_{\mu\nu})^a + g_s \overline{q}\gamma_\mu T^a q = 0. \tag{2.7}$$

Thus, by use of EOM, this operator is rewritten by 4-Fermi operator as

$$\mathcal{O}_{qG} = -ig_s(\overline{q}\gamma^\mu T^a q)(\overline{q}'\gamma^\mu T^a q') \equiv -ig_s\mathcal{O}_{4F}. \tag{2.8}$$

It means that $\mathcal{O}_{qG}$ is not independent from $\mathcal{O}_{4F}$ anymore through the EOMs.* This is a way to obtain a minimal "complete" set of dimension six operators in Ref.[4] by reduction of redundant operators through EOMs. However, the use of EOMs must need more careful treatment, and actually, we must not use EOMs in a calculation of a scattering process (S-matrix element). We will explain its validity by using a process, $gg \to qq$, for example. For this process, dimension six operators which include interactions of quark-quark-gluon and quark-quark-gluon-gluon must contribute as in Fig.1. Among a complete set in Ref[2], $\mathcal{O}_{qG}$ is the only operator which has the

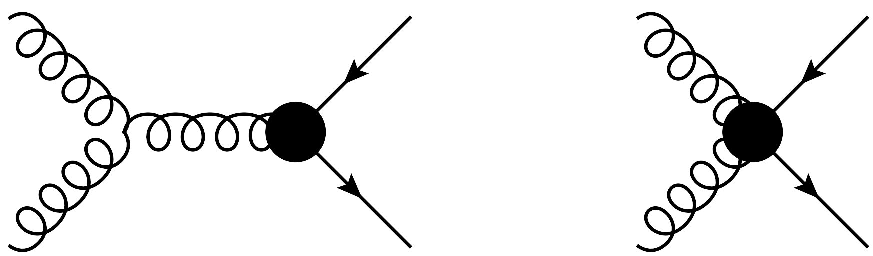

Figure 1: Scattering processes of $gg \to qq$: Black circles denote a vertex of the dimension six operator. There surely exist two diagrams which include the irrelevant operator, however, these diagram might happen to vanish if we use EOMs.

interactions as

$$\mathcal{O}_{qG} \supset (qqG),\ (qqGG),\ (qqGGG). \tag{2.9}$$

Thus, a calculation of $\langle qq|\mathcal{O}_{qG}|gg\rangle$ is seemed to be enough for our goal. However, $\mathcal{O}_{qG}$ is rewritten by 4-Fermi operator by use of EOMs as shown above, so that matrix element of $\langle qq|\mathcal{O}_{qG}|gg\rangle$ might happen to vanish. This conclusion is quit suspicious, and we must be careful to deal with EOMs in a calculation of matrix element with general irrelevant operators as will be also shown in Appendix A. We must need the accurate calculation of irrelevant operators without use of EOMs. In this paper we suggest a calculation method where we do not use EOMs, and calculate concrete dimension six effective operators by use of some new physics as the underlying theories beyond the SM.

We now stand in a position of showing our accurate method of calculating irrelevant operators without use of EOMs. Let us show a concrete calculation by using a toy model. We consider a Lagrangian,

$$\mathcal{L} = \overline{\psi}_l(i\partial\!\!\!/ - m)\psi_l + \overline{\psi}_h(i\partial\!\!\!/ - M)\psi_h + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}M^2\phi^2 - g\phi(\overline{\psi}_l\psi_h + \overline{\psi}_h\psi_l), \tag{2.10}$$

* There are of cause other 4-Fermi operators.

where ψ_l (ψ_h) denotes a light (heavy) Dirac fermion and ϕ is a heavy real scalar with $m \ll M$. We will obtain an effective action of ψ_l after integrating out heavy fields, where irrelevant operators must include traces of heavy particles and their interactions at high energy scale. Let us calculate the effective action by integrate out ψ_h, ϕ, and show dimension six operators by expanding $1/M^n$. The effective action should be given by

$$e^{iS_{\text{eff}}[\psi_l]} = \int \mathcal{D}\phi \mathcal{D}\psi_h \mathcal{D}\overline{\psi}_h e^{iS[\psi_l,\psi_h,\phi]}, \tag{2.11}$$

and firstly, by integrating out H, it becomes

$$\begin{aligned} &= \int \mathcal{D}\phi \mathcal{D}\psi_h \mathcal{D}\overline{\psi}_h \exp i \left\{ S_{\text{free}}[\psi_l, \phi] + (\overline{\psi}_h - \overline{A}K_0^{-1})K_0(\psi_h - K_0^{-1}A) - \overline{A}K_0^{-1}A \right\}, \\ &= (\text{Det}K_0) \exp i \left\{ S_{\text{free}}[\psi_l, \phi] - \overline{A}K_0^{-1}A \right\}, \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} &K_0^{-1} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{\not{p} - M} e^{-ip(x-y)} = -i\mathcal{D}^{(\psi_h)}(x-y), \\ &A = g\phi\psi_l, \ \ \overline{A} = g\overline{\psi}_l\phi. \end{aligned} \tag{2.13}$$

The second term in Eq.(2.12) is written by

$$-\overline{A}K_0^{-1}A = -\frac{1}{2} \int d^4x d^4y \phi(x) \delta\tilde{K}(x,y)\phi(y) \equiv -\frac{1}{2}\phi \cdot \delta\tilde{K} \cdot \phi, \tag{2.14}$$

$$\delta\tilde{K}(x,y) \equiv -2ig^2\overline{\psi}_l(x)\mathcal{D}^{(\psi_h)}(x-y)\psi_l(y). \tag{2.15}$$

Next step is an integration of ϕ, which gives

$$\begin{aligned} e^{iS_{\text{eff}}[\psi_l]} &= (\text{Det}K_0) \int \mathcal{D}\phi \exp i \left\{ S_{\text{free}}[\psi_l] - \frac{1}{2}\phi \cdot (\tilde{K}_0 + \delta\tilde{K}) \cdot \phi \right\}, \\ &= \left(\frac{\text{Det}K_0}{\text{Det}^{\frac{1}{2}}(\tilde{K}_0 + \delta\tilde{K})} \right) e^{iS_{\text{free}}[\psi_l]}, \\ &= \left(\frac{\text{Det}K_0}{\text{Det}^{\frac{1}{2}}\tilde{K}_0} \right) \exp \left\{ iS_{\text{free}}[\psi_l] - \frac{1}{2}\text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\tilde{K}_0^{-1}\delta\tilde{K})^n \right\}, \end{aligned} \tag{2.16}$$

$$\tilde{K}_0^{-1}(x,y) \equiv \int \frac{d^4p}{(2\pi)^4} \frac{-1}{p^2 - M^2} e^{-ip(x-y)} = i\mathcal{D}^{(\phi)}(x-y). \tag{2.17}$$

Determinant of K_0 and $\tilde{K}_0$ are cancelled by normalization, so that we finally obtain the effective action of ψ_l as

$$S_{\text{eff}}[\psi_l] = S_{\text{free}}[\psi_l] + \frac{i}{2}\text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\tilde{K}_0^{-1}\delta\tilde{K})^n. \tag{2.18}$$

Higher dimensional operators are included in the second term of Eq.(2.18), thus dimension six operators are calculated from the second order of $1/M^n$ expansion. A space integration of $\mathcal{O}(1/M^2)$ gives

$$
\begin{aligned}
&-\frac{i}{4}\int d^4xd^4yd^4zd^4w\tilde{K}_0^{-1}(x,y)\delta\tilde{K}(y,z)\tilde{K}_0^{-1}(z,w)\delta\tilde{K}(w,x),\\
&=-ig^4\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(k_1-k_2+k_3-k_4),\\
&\times\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-M^2}\overline{\psi}_l(k_1)\frac{1}{\not{p}+\not{k}_1-M}\psi_l(k_2)\frac{1}{(p+k_1-k_2)^2-M^2}\overline{\psi}_l(k_2)\frac{1}{\not{p}+\not{k}_4-M}\psi_l(k_4),
\end{aligned}
\tag{2.19}
$$

and integration of all momenta $p, k_i, (i=1,\cdots,4)$ with $k_i \ll M$ induces 4-Fermi operators,

$$
\mathcal{O}_{4\mathrm{F}}(x)=\frac{g^4}{192\pi^2}\frac{1}{M^2}\left[-(\overline{\psi}_l\gamma^\mu\psi_l)(\overline{\psi}_l\gamma_\mu\psi_l)+2(\overline{\psi}_l\psi_l)(\overline{\psi}_l\psi_l)\right]. \tag{2.20}
$$

These are the dimension six operators in this model. Notice that accurate coefficients are automatically obtained without care of symmetric factors. Other higher order operators can be calculated similarly.

3 Dimension six operators induced from new physics

By using of the calculation method represented in the previous section, we concretely calculate coefficients of dimension six operators induced from some candidates of new physics, SUSY, UED, and LH. We obtain coefficients of dimension six operators in QCD by integrating out SUSY and UED particles. We also obtain coefficients of 4-Fermi operators originating from anomalous interactions induced from LH.

3.1 SUSY

Let us calculate coefficients of dimension six operators in QCD when beyond the SM is SUSY. In the SUSY with R-parity, SUSY particles propagate only inside of loop diagrams. Lagrangian of the QCD sector in SUSY SM is given by

$$
\begin{aligned}
\mathcal{L}=&\mathcal{L}_{SM}+\frac{1}{2}\overline{\tilde{g}}(i\not{\partial}-m_{\tilde{g}})\tilde{g}-\tilde{q}_R^\dagger(\partial^2+m_{\tilde{q}_R}^2)\tilde{q}_R-\tilde{q}_L^\dagger(\partial^2+m_{\tilde{q}_L}^2)\tilde{q}_L\\
&+\frac{i}{2}g_sf^{abc}\overline{\tilde{g}^a}\gamma^\mu\tilde{g}^bG_\mu^c-ig_s\sum_q(\tilde{q}_L^\dagger\frac{\lambda^a}{2}\overleftrightarrow{\partial}^\mu\tilde{q}_L)G_\mu^a-ig_s\sum_q(\tilde{q}_R^\dagger\frac{\lambda^a}{2}\overleftrightarrow{\partial}^\mu\tilde{q}_R)G_\mu^a\\
&+g_s^2\sum_q\tilde{q}_L^\dagger\frac{\lambda^a}{2}\frac{\lambda^b}{2}\tilde{q}_LG_\mu^aG^{b\mu}+g_s^2\sum_q\tilde{q}_R^\dagger\frac{\lambda^a}{2}\frac{\lambda^b}{2}\tilde{q}_RG_\mu^aG^{b\mu}\\
&-\sqrt{2}g_s\sum_q(\tilde{q}_L^\dagger\overline{\tilde{g}_R^a}\frac{\lambda^a}{2}q_L+\mathrm{h.c.})+\sqrt{2}g_s\sum_q(\tilde{q}_R^\dagger\overline{\tilde{g}_L^a}\frac{\lambda^a}{2}q_R+\mathrm{h.c.}).
\end{aligned}
\tag{3.21}
$$

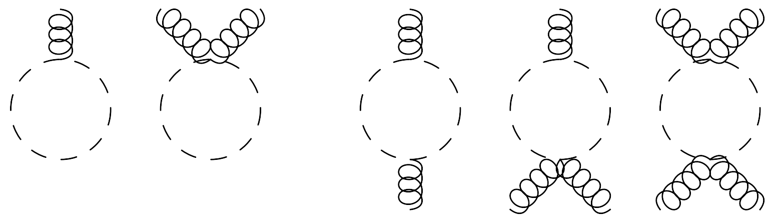

Figure 2: Diagrams of gluon external lines.

where $\sum_q$ represents a sum over all flavors. Effective action should be obtained by integrating out $\tilde{q}_L, \tilde{q}_R$, and $\tilde{g}$ as

$$e^{iS_{\rm eff}} = \int \mathcal{D}\tilde{g}\mathcal{D}\tilde{q}_L\mathcal{D}\overline{\tilde{q}_L}\mathcal{D}\tilde{q}_R\mathcal{D}\overline{\tilde{q}_R}\ e^{iS}. \tag{3.22}$$

$S_{\rm eff}$ includes all possible irrelevant operators. Calculating results are listed in Appendix C.1, where coefficients of dimension six operators, 4-Fermi $\mathcal{O}_{qqqq}$, quark-quark-gluon-gluon $\mathcal{O}_{qqGG}$, and quark-quark-gluon $\mathcal{O}_{qqG}$ are represented. We overview explicit technique to calculate them in the following discussions.

The first step is integrating out $\tilde{q}_R$ as

$$\int \mathcal{D}\tilde{q}_R\mathcal{D}\overline{\tilde{q}_R}e^{iS} = \exp i\ [i\mathrm{Tr}(\log K) + B^\dagger K^{-1}B], \tag{3.23}$$

$$\begin{array}{ll} K = K_0 + \delta K, & \\ K_0 = (\partial^\mu\partial_\mu + m_R^2), & \delta K = ig_s[2G_\mu\partial^\mu + (\partial^\mu G_\mu)] - g_s^2 G_\mu G^\mu, \\ B = -\sqrt{2}g_s(\overline{\tilde{g}^a}\frac{\lambda}{2}^a P_R q), & B^\dagger = -\sqrt{2}g_s(\overline{q}P_L\frac{\lambda}{2}^a\tilde{g}^a), \end{array} \tag{3.24}$$

where $\mathrm{Tr}(\log K)$ includes some loop diagrams which have external gluon lines (e.g Fig. 2). The second term, $B^\dagger K^{-1}B$, can be expanded by right-handed squark propagator as

$$\begin{aligned} B^\dagger K^{-1}B &= B^\dagger(1 + K_0^{-1}\delta K)^{-1}K_0^{-1}B \\ &= B^\dagger K_0^{-1}B - B^\dagger K_0^{-1}\delta K K_0^{-1}B + B^\dagger K_0^{-1}\delta K K_0^{-1}\delta K K_0^{-1}B + \cdots . \end{aligned} \tag{3.25}$$

We take the expansion up to order of g_s^4 in $S_{\rm eff}$. Similarly, $\tilde{q}_L$ integration can be performed, and after $\tilde{q}_R, \tilde{q}_L$ integrations, an "effective action" in this stage is given by

$$S^{(\tilde{g})} = \frac{1}{2}\int d^4x d^4y \overline{\tilde{g}}(x)_i^a \left[\tilde{K}_0 + \tilde{K}_I\right]_{xy}^{ab} \tilde{g}(y)_j^b, \tag{3.26}$$

where $\tilde{K}_0$ and $\tilde{K}_I$ are

$$\left[\tilde{K}_0\right]_{xy}^{ab} \equiv \delta^{ab}\delta^4(x-y)(i\partial\!\!\!/_y - m_{\tilde{g}})_{ij}, \tag{3.27}$$

$$
\begin{aligned}
\left[\tilde{K}_I\right]^{ab}_{xy} &\equiv ig_s f^{abc}\delta^4(x-y)\not{G}^c_{ij} - 4ig_s^2 \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda}{2}^a q_i(x) \\
&- 4ig_s^3 \int d^4z \sum_{q=q_L,q_R} \overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-x)\frac{\lambda}{2}^a q_i(x) \\
&- 4ig_s^4 \int d^4z d^4w \sum_{q=q_L,q_R} \overline{q}_j(y)\overline{q}_j(y)\frac{\lambda}{2}^b \mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-w) \\
&\cdot \{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_w \mathcal{D}^{(\tilde{q})}(w-x)\frac{\lambda}{2}^a q_i(x) + \mathcal{O}(g_s^5),
\end{aligned} \tag{3.28}
$$

and

$$
\mathcal{D}^{(\tilde{q})}(x-y) = -iK_0^{-1} = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m_{\tilde{q}}^2 + i\epsilon}\mathrm{e}^{-ik\cdot(x-y)}. \tag{3.29}
$$

Here i, j denote spinor indexes. Next step is integrating out gluino, and the final effective action is obtained as

$$
\begin{aligned}
S_{\text{eff}} = &\int d^4x d^4y \alpha(x,y)^{ab}_{ij}\tilde{K}_I(x,y)^{ab}_{ij} \\
&+ \int d^4x d^4y d^4z d^4w \beta(x,y,z,w)^{abcd}_{ijkl}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z.w)^{cd}_{kl} + \mathcal{O}(\tilde{K}_I^3),
\end{aligned} \tag{3.30}
$$

where $\alpha(x,y),\ \beta(x,y,z,w)$ consist of gluino propagator as

$$
\alpha(x,y)^{ab}_{ij} = -\frac{1}{2}\delta^{ab}\mathcal{D}^{(\tilde{g})}(y-x)_{ji}, \tag{3.31}
$$

$$
\begin{aligned}
&\beta(x,y,z,w)^{abcd}_{ijkl} \\
&= \frac{i}{8}\delta^{ac}\delta^{bd}\left[C^\dagger\mathcal{D}^{(\tilde{g})}(x-z)\right]_{ik}\left[\mathcal{D}^{(\tilde{g})}(w-y)C^T\right]_{lj} - \frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj}.
\end{aligned} \tag{3.32}
$$

We can know these results by differentiating interacting parts of the effective action as

$$
\left.\frac{\delta}{\delta\tilde{K}_I(x,y)^{ab}_{ij}}e^{iS_{\text{eff}}}\right|_{\tilde{K}_I=0} = i\alpha(x,y)^{ab}_{ij}, \tag{3.33}
$$

$$
\begin{aligned}
\left.\frac{\delta}{\delta\tilde{K}_I(z,w)^{cd}_{kl}}\frac{\delta}{\delta\tilde{K}_I(x,y)^{ab}_{ij}}e^{iS_{\text{eff}}}\right|_{\tilde{K}_I=0} &= -\alpha(x,y)^{ab}_{ij}\alpha(z,w)^{cd}_{kl} \\
&+ i\beta(x,y,z.w)^{abcd}_{ijkl} + i\beta(z,w,x,y)^{cdab}_{klij}.
\end{aligned} \tag{3.34}
$$

In this stage, all $\mathcal{O}_{qqqq}$ and $\mathcal{O}_{qqG}$ at 1-loop level are included in Eq.(3.30) up to the second order of $\tilde{K}_I$.

For example, the 4-Fermi operators all $\mathcal{O}_{qqqq}$ are obtained by picking up $\mathcal{O}(g_s^2)$ order terms

from each $\tilde{K}_I$ in $\beta\tilde{K}_I\tilde{K}_I$, which is given by

$$\int d^4xd^4yd^4zd^4w\beta(x,y,z,w)^{abcd}_{ijkl}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z.w)^{cd}_{kl}$$
$$\supset \int d^4xd^4yd^4zd^4w\left[\frac{i}{8}\delta^{ac}\delta^{bd}\left[C^\dagger\mathcal{D}^{(\tilde{g})}(x-z)\right]_{ik}\left[\mathcal{D}^{(\tilde{g})}(w-y)C^T\right]_{lj}\right.$$
$$\left.-\frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj}\right]$$
$$\times\left[-4ig_s^2\overline{q}_j(y)\frac{\lambda}{2}^b\mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda}{2}^a q_i(x)\right]\cdot\left[-4ig_s^2\overline{q}'_l(w)\frac{\lambda}{2}^d\mathcal{D}^{(\tilde{q}')}(w-z)\frac{\lambda}{2}^c q'_k(z)\right]. \tag{3.35}$$

The first term is given by

$$-2ig_s^4\int d^4xd^4yd^4zd^4w\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{d^4p_3}{(2\pi)^4}\frac{d^4p_4}{(2\pi)^4}$$
$$\times\left[\overline{q}_j(k_2)\frac{\lambda}{2}^b\frac{\lambda}{2}^a q_i(k_1)\right]\cdot\left[\overline{q}'_l(k_4)\frac{\lambda}{2}^d\frac{\lambda}{2}^c q'_k(k_3)\right]\frac{i[C^\dagger(\not{p}_1+m_{\tilde{g}})]_{ik}}{p_1^2-m_{\tilde{g}}^2}\frac{i[(\not{p}_2+m_{\tilde{g}})C^T]_{lj}}{p_2^2-m_{\tilde{g}}^2}\frac{i}{p_3^2-m_{\tilde{q}}^2}\frac{i}{p_4^2-m_{\tilde{q}'}^2}$$
$$\times e^{ik_2y}e^{-ik_1x}e^{ik_4x}e^{-ik_3z}e^{-ip_1(x-z)}e^{ip_2(w-y)}e^{-ip_3(y-x)}e^{-ip_4(w-z)}, \tag{3.36}$$

and by integrating out $x, y, z, w, p_2, p_3, p_4$, this term becomes

$$=-2ig_s^4\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}\delta^4(k_1-k_2+k_3-k_4)$$
$$\times\left[\overline{q}_j(k_2)\frac{\lambda}{2}^b\frac{\lambda}{2}^a q_i(k_1)\right]\cdot\left[\overline{q}'_l(k_4)\frac{\lambda}{2}^d\frac{\lambda}{2}^c q'_k(k_3)\right]\left[A(C^\dagger\gamma^\mu)_{ik}(\gamma^\nu C^T)_{lj}+B(C^\dagger)_{ik}(C^T)_{lj}\right], \tag{3.37}$$
$$A=\int\frac{d^4p_1}{(2\pi)^4}\frac{p_{1\mu}(p_1+k_1-k_2)_\nu}{(p_1^2-m_{\tilde{g}}^2)[(p_1+k_1-k_2)^2-m_{\tilde{g}}^2][(p_1+k_1)^2-m_{\tilde{q}}^2][(p_1-k_1)^2-m_{\tilde{q}'}^2]}, \tag{3.38}$$
$$B=\int\frac{d^4p_1}{(2\pi)^4}\frac{m_{\tilde{g}}^2}{(p_1^2-m_{\tilde{g}}^2)[(p_1+k_1-k_2)^2-m_{\tilde{g}}^2][(p_1+k_1)^2-m_{\tilde{q}}^2][(p_1-k_1)^2-m_{\tilde{q}'}^2]}. \tag{3.39}$$

Here A, B are Feynman parameter integral, and they become

$$A\to -i\frac{6}{192\pi^2}f_1(m_{\tilde{q}},m_{\tilde{q}}),\quad B\to i\frac{12}{192\pi^2}f_2(m_{\tilde{q}},m_{\tilde{q}}), \tag{3.40}$$

when $k_i, (i=1,\cdots,4)$ are much smaller than masses of squarks and gluino. f_1, f_2 are shown in Appendix C.1, and the spinor can be rearranged by Fierz transformation of Eqs.(B.69) and (B.70) in Appendix B. Necessary Fierz transformations and color factors are shown in Appendix B.

We can summarize all 4-Fermi operators as separating color singlet $\mathcal{O}^{(1)}_{qqqq}$ or color octet $\mathcal{O}^{(8)}_{qqqq}$, which is shown in Appendix C.1.1. When their chiralities are $(LL)(LL)$ or $(RR)(RR)$ (L:

left-handed, R: right-handed), the 4-Fermi operators are given by

$$\mathcal{O}^{(1)}_{qqqq} = \frac{12}{192\pi^2} g_s^4 \left[\frac{2}{9}(f_1 + f_2) \right] (\overline{q}\gamma^\mu q)(\overline{q}'\gamma^\mu q'), \tag{3.41}$$

$$\mathcal{O}^{(8)}_{qqqq} = \frac{12}{192\pi^2} g_s^4 \left[-\frac{1}{3} f_1 - \frac{7}{6} f_2 \right] (\overline{q}\gamma^\mu \frac{\lambda^a}{2} q)(\overline{q}'\gamma^\mu \frac{\lambda^a}{2} q'). \tag{3.42}$$

On the other hand, when their chiralities are $(LL)(RR)$ or $(RR)(LL)$, the 4-Fermi operators are given by

$$\mathcal{O}^{(1)}_{qqqq} = \frac{12}{192\pi^2} g_s^4 \left[\frac{2}{9}(-f_1 + f_2) \right] (\overline{q}\gamma^\mu q)(\overline{q}'\gamma^\mu q'), \tag{3.43}$$

$$\mathcal{O}^{(8)}_{qqqq} = \frac{12}{192\pi^2} g_s^4 \left[-\frac{7}{6} f_1 - \frac{1}{3} f_2 \right] (\overline{q}\gamma^\mu \frac{\lambda^a}{2} q)(\overline{q}'\gamma^\mu \frac{\lambda^a}{2} q'). \tag{3.44}$$

As for $\mathcal{O}_{qqG}$, vertex originates from two parts, $\alpha\tilde{K}_I$ and $\beta\tilde{K}_I\tilde{K}_I$, as

$$\begin{aligned}
&\int d^4x d^4y \alpha(x,y)^{ab}_{ij} \tilde{K}_I(x,y)^{ab}_{ij} \supset \int d^4x d^4y \left[-\frac{1}{2}\delta^{ab}\mathcal{D}^{(\tilde{g})}(y-x)_{ji} \right] \\
&\times \left[-4ig_s^3 \int d^4z \sum_{q=q_L,q_R} \overline{q}_j(y) \frac{\lambda^b}{2} \mathcal{D}^{(\tilde{q})}(y-z)\{2G_\mu\partial^\mu + (\partial^\mu G_\mu)\}_z \mathcal{D}^{(\tilde{q})}(z-x) \frac{\lambda^a}{2} q_i(x) \right],
\end{aligned} \tag{3.45}$$

$$\begin{aligned}
&\int d^4x d^4y d^4z d^4w \beta(x,y,z,w)^{abcd}_{ijkl} \tilde{K}_I(x,y)^{ab}_{ij} \tilde{K}_I(z.w)^{cd}_{kl} \\
&\supset 2 \int d^4x d^4y d^4z d^4w \left[\frac{i}{8}\delta^{ac}\delta^{bd} \left[C^\dagger \mathcal{D}^{(\tilde{g})}(x-z) \right]_{ik} \left[\mathcal{D}^{(\tilde{g})}(w-y) C^T \right]_{lj} \right. \\
&\left. -\frac{i}{8}\delta^{ad}\delta^{bc}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-z)_{lj} \right] \\
&\times \left[ig_s f^{abc}\delta^4(x-y) \not{G}^c_{ij} \right] \cdot \left[-4ig_s^2 \sum_{q=q_L,q_R} \overline{q}_j(y) \frac{\lambda^b}{2} \mathcal{D}^{(\tilde{q})}(y-x) \frac{\lambda^a}{2} q_i(x) \right].
\end{aligned} \tag{3.46}$$

Equations (3.45) and (3.46) include not only dimension four operators but also all higher dimensional operators such as dimension six operator. Higher dimensional operators have been obtained by expanding the full operator by $k^2 \ll \Lambda^2$, where k^μ and Λ denote the momentum of the SM particles and SUSY particles, respectively. Anyhow, we can obtain all $\mathcal{O}_{qqG}$ in the similar calculations as 4-Fermi operators, which is shown Appendix C.1.2.

For $\mathcal{O}_{qqGG}$, they can be also obtained in the same manner. $\mathcal{O}_{qqGG}$ contains in Eq.(3.30), and there are two contributions in the first order of $\tilde{K}_I$ as

$$\begin{aligned}
&\int d^4x d^4y \alpha(x,y)^{ab}_{ij} \tilde{K}_I(x,y)^{ab}_{ij} \\
&\supset \int d^4x d^4y \alpha(x,y)^{ab}_{ij} \cdot 4g_s^2 \int d^4z \overline{q}_j(y) \frac{\lambda^a}{2} i\mathcal{D}^{(\tilde{q})}(y-z)[-g_s^2 G_\mu G^\mu]_z i\mathcal{D}^{(\tilde{q})}(z-x) \frac{\lambda^a}{2} q_i(x),
\end{aligned} \tag{3.47}$$

$$
\begin{aligned}
&\int d^4xd^4y\alpha(x,y)^{ab}_{ij}\tilde{K}_I(x,y)^{ab}_{ij} \\
&\quad\supset \int d^4xd^4yd^4zd^4w\alpha(x,y)^{ab}_{ij}(-4g_s^2)\overline{q}_j(y)\frac{\lambda}{2}^a i\mathcal{D}^{(\tilde{q})}(y-z)ig_s[2G^\mu\partial_\mu+(\partial_\mu G^\mu)]_z \\
&\quad\times i\mathcal{D}^{(\tilde{q})}(z-w)ig_s[2G^\mu\partial_\mu+(\partial_\mu G^\mu)]_w i\mathcal{D}^{(\tilde{q})}(w-x)\frac{\lambda}{2}^a q_i(x).
\end{aligned} \tag{3.48}
$$

Similarly, there is one contribution in the second order of $\tilde{K}_I$, which is shown as

$$
\begin{aligned}
&\int d^4xd^4yd^4zd^4w\beta(x,y,z,w)^{abcd}_{ijkl}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z.w)^{cd}_{kl} \\
&\quad\supset 2\int d^4xd^4yd^4zd^4w\beta(x,y,z,w)^{abcd}_{ijkl}[ig_sf^{abe}\not{G}^e_{ij}(x)\delta^4(x-y)] \\
&\quad\times\int d^4z'4g_s^2\overline{q}_l(z)\frac{\lambda}{2}^d i\mathcal{D}^{(\tilde{q})}(z-z')\{ig_s(2G^\mu\partial_\mu+(\partial_\mu G^\mu)\}_{z'}i\mathcal{D}^{(\tilde{q})}(z'-w)\frac{\lambda}{2}^c q_k(w)
\end{aligned} \tag{3.49}
$$

There is one contribution in the third order in Eq.(3.30), $\gamma\tilde{K}_I\tilde{K}_I\tilde{K}_I$, where γ is given by

$$
\begin{aligned}
\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn} = \frac{1}{8}\big\{&\delta^{ad}\delta^{be}\delta^{cf}\mathcal{D}^{(\tilde{g})}(w-x)_{li}\mathcal{D}^{(\tilde{g})}(y-u)_{jm}\mathcal{D}^{(\tilde{g})}(v-z)_{nk} \\
&+\delta^{af}\delta^{bc}\delta^{ed}\mathcal{D}^{(\tilde{g})}(v-x)_{ni}\mathcal{D}^{(\tilde{g})}(y-x)_{jk}\mathcal{D}^{(\tilde{g})}(w-u)_{lm} \\
&-\delta^{ac}\delta^{be}\delta^{df}[C^\dagger\mathcal{D}^{(\tilde{g})}(z-x)]_{ki}\mathcal{D}^{(\tilde{g})}(y-u)_{jm}[\mathcal{D}^{(\tilde{g})}(v-w)C^T]_{nl} \\
&-\delta^{ac}\delta^{bf}\delta^{de}[C^\dagger\mathcal{D}^{(\tilde{g})}(x-z)]_{ik}[\mathcal{D}^{(\tilde{g})}(v-y)C^T]_{nj}\mathcal{D}^{(\tilde{g})}(w-u)_{lm} \\
&-\delta^{ad}\delta^{bf}\delta^{ce}\mathcal{D}^{(\tilde{g})}(w-x)_{li}[\mathcal{D}^{(\tilde{g})}(y-v)C^T]_{jn}[C^\dagger\mathcal{D}^{(\tilde{g})}(u-z)]_{mk} \\
&-\delta^{ae}\delta^{bc}\delta^{fd}[C^\dagger\mathcal{D}^{(\tilde{g})}(u-x)]_{mi}\mathcal{D}^{(\tilde{g})}(y-z)_{jk}[\mathcal{D}^{(\tilde{g})}(w-v)C^T]_{ln} \\
&-\delta^{ae}\delta^{bd}\delta^{fc}[C^\dagger\mathcal{D}^{(\tilde{g})}(x-u)]_{im}[\mathcal{D}^{(\tilde{g})}(y-w)C^T]_{lj}\mathcal{D}^{(\tilde{g})}(v-z)_{nk} \\
&-\delta^{af}\delta^{bd}\delta^{ec}\mathcal{D}^{(\tilde{g})}(v-x)_{ni}[\mathcal{D}^{(\tilde{g})}(y-w)C^T]_{jl}[C^\dagger\mathcal{D}^{(\tilde{g})}(z-u)]_{km}\big\}\,,
\end{aligned} \tag{3.50}
$$

and the third order is shown as

$$
\begin{aligned}
&\int d^4x\cdots d^4v\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn}\tilde{K}_I(x,y)^{ab}_{ij}\tilde{K}_I(z,w)^{cd}_{kl}\tilde{K}_I(u,v)^{ef}_{mn} \\
&\quad\supset\int d^4x\cdots d^4v\gamma(x,y,z,w,u,v)^{abcdef}_{ijklmn}\left[(-4g_s^2)\overline{q}_j(y)\frac{\lambda}{2}^b i\mathcal{D}^{(\tilde{q})}(y-x)\frac{\lambda}{2}^a q_i(x)\right] \\
&\quad\times\left[ig_sf^{cdg}\not{G}^g(z)_{kl}\delta^4(z-w)\right]\left[ig_sf^{efh}\not{G}^h(u)_{mn}\delta^4(u-v)\right].
\end{aligned} \tag{3.51}
$$

Although γ has eight terms in total, they are all the same in Eq.(3.51) since each term of γ corresponds to statistic factor in Feynman diagram. Notice again that we do not care about a statistic factor in each operator since it is automatically included in α, β, γ. We can obtain all $\mathcal{O}_{qqGG}$ induced from SUSY which is shown Appendix C.1.3.

3.2 UED

Next, we estimate QCD dimension six operators induced from UED. The UED has KK-parity so that KK particles can propagate only inside loop processes. As the SUSY case, we can calculate dimension six operators by integrating out KK particles.

After dimensional reduction of the fifth dimensional compactified space of S^1/Z_2, UED Lagrangian in the 4-dimensional space-time is given by

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{q^{(n)}} + \mathcal{L}_{G^{(n)}} + \mathcal{L}_{G_5^{(n)}},$$

$$\begin{aligned}
\mathcal{L}_{q^{(n)}} = & \sum_n \Big[\overline{q_L}^{(n)} i(\partial\!\!\!/ + ig_s G\!\!\!/ - m_L^{(n)})q_L^{(n)} + \overline{q_R}^{(n)} i(\partial\!\!\!/ + ig_s G\!\!\!/ + m_R^{(n)})q_R^{(n)} \\
& -g_s(\overline{q} G\!\!\!/^{(n)} P_L q^{(n)} + \overline{q} i\gamma^5 G_5^{(n)} P_R q^{(n)}) - (L \longleftrightarrow R)\Big] + \cdots ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{G^{(n)}} = & \sum_n \Big[-\frac{1}{4}(\partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a})^2 + \frac{1}{2} m_g^{(n)2} G_\mu^{(n)a} G^{(n)a\mu} \\
& -\tfrac{1}{2} g_s f^{abc}(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) G^{(n)b\mu} G^{(n)c\nu} - \tfrac{1}{2} g_s f^{abc}(\partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a}) G^{(n)b\mu} G^{c\nu} \\
& -\tfrac{1}{2} g_s f^{abc} \partial_\mu G_\nu^{(n)a} - \partial_\nu G_\mu^{(n)a}) G^{b\mu} G^{(n)c\nu} - \tfrac{1}{4} f^{abc} f^{ade} \Big\{ 2G_\mu^{(0)b} G_\nu^{(0)c} G^{(n)d\mu} G^{(n)e\mu} \\
& +(G_\mu^{(0)b} G_\nu^{(n)c} + G_\mu^{(n)b} G_\nu^{(0)c})(G^{(0)d\mu} G^{(n)e\mu} + G^{(n)d\mu} G^{(0)e\mu})\Big\}\Big] \cdots ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{G_5^{(n)}} = & \sum_n \Big[\frac{1}{2} \partial_\mu G_5^{(n)a} \partial^\mu G_5^{(n)a} - \frac{1}{2} m_5^2 G_5^{(n)a} G_5^{(n)a} \\
& + g_s f^{abc}(m_g G_\nu^{(n)a} + \partial_\nu G_5^{(n)a}) G^{(n)b5} G^{c\nu} - \tfrac{1}{2} g_s^2 f^{abc} f^{ade} G_5^{(n)b} G_\nu^c G^{(n)d5} G^{e\nu}\Big] + \cdots ,
\end{aligned}$$

where

$$\begin{aligned}
m_L^{(n)} &= \frac{n}{R} + \delta m_L, \quad m_R^{(n)} = \frac{n}{R} + \delta m_R, \\
m_g^{(n)} &= \frac{n}{R} + \delta m_g, \quad m_5^{(n)} = \frac{n}{R} + \delta m_5.
\end{aligned}$$

Here we take a 'tHooft-Feynman gauge fixing, and $m_L^{(n)}, m_R^{(n)}, m_g^{(n)}, m_5^{(n)}$ are $SU(2)$ doublet KK quark mass, $SU(2)$ singlet KK quark mass, KK gluon mass, and KK scalar (fifth dimensional component of KK gluon) mass with each radiative correction, respectively. At a tree level, these KK particles are degenerate in a minimal UED, but there is a slight difference between $m_L^{(n)}$ and $m_R^{(n)}$ when we consider radiative corrections. So here we show general effective operators by using these general parameters. The effective operators of S_{eff} in UED can be calculated by the similar technique of the previous subsection, and the results are shown in Appendix C.2.

Here we overview this calculation. By integrating out KK quarks, KK scalars, and KK gluons, the effective action becomes

$$\begin{aligned}
S_{\text{eff}} = \tilde{S} + & \int d^4x d^4y \alpha(x,y)_{\mu\nu}^{ab} K_I(x,y)_{\mu\nu}^{ab} \\
& + \int d^4x d^4y d^4z d^4w \beta(x,y,z,w)_{\mu\nu\rho\sigma}^{abcd} K_I(x,y)_{\mu\nu}^{ab} K_I(z,w)_{\rho\sigma}^{cd} + \cdots , \qquad (3.52)
\end{aligned}$$

where $\tilde{S}$ does not include KK gluons that is given by

$$\begin{aligned}
\tilde{S} &= -g_s^3 \int d^4x d^4y d^4z \mathcal{D}^{(s)}(x-y) \left[\overline{q}_L(x) \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \not{G}(z) \mathcal{D}^{(L)}(z-y) \frac{\lambda}{2}^a q_L(y) + (L \leftrightarrow R) \right] \\
&+ i \int d^4x d^4y d^4z_1 d^4z_2 d^4z_3 \delta^4(x-y) \\
&\times \left[g_s f^{bcf} G^{f\mu}(z_1) \partial_{z_1\mu} \delta^4(z_1 - z_2) + i g_s^2 \left\{ \overline{q}_L(z_1) \frac{\lambda}{2}^b \mathcal{D}^{(L)}(z_1 - z_2) \frac{\lambda}{2}^c q_L(z) + (L \leftrightarrow R) \right\} \right] \\
&\times \left[\mathcal{D}^{(s)}(x - z_1) \delta^{ab} \right] \\
&\times \left[g_s f^{deg} G^{g\nu}(z_3) \partial_{z_3\nu} \delta^4(z_3 - y) + i g_s^2 \left\{ \overline{q}_L(z_3) \frac{\lambda}{2}^b \mathcal{D}^{(L)}(z_3 - y) \frac{\lambda}{2}^c q_L(y) + (L \leftrightarrow R) \right\} \right] \\
&\times \left[\mathcal{D}^{(s)}(z_2 - z_3) \delta^{cd} \right]. \qquad (3.53)
\end{aligned}$$

Here $\mathcal{D}^{(L)}$, $\mathcal{D}^{(R)}$, and $\mathcal{D}^{(s)}$ are propagators of KK quarks and KK scalars as

$$\mathcal{D}^{(L)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} - m_L^{(n)}} e^{-ip(x-y)}, \qquad (3.54)$$

$$\mathcal{D}^{(R)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{\not{p} + m_R^{(n)}} e^{-ip(x-y)}, \qquad (3.55)$$

$$\delta^{ab} \mathcal{D}^{(s)} = \delta^{ab} \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m_5^{(n)2}} e^{-ip(x-y)}, \qquad (3.56)$$

respectively. And α, β, K_I are given by

$$\alpha(x,y)^{ab}_{\mu\nu} = \frac{1}{2} \mathcal{D}^{(g)}(x-y) \delta^{ab} g_{\mu\nu} \equiv \frac{1}{2} \delta^{ab} g_{\mu\nu} \int \frac{d^4p}{(2\pi)^4} \frac{-i}{p^2 - m_g^2} e^{-i(x-y)}, \qquad (3.57)$$

$$\begin{aligned}
&\beta(x,y,z,w)^{abcd}_{\mu\nu\rho\sigma} \\
&= \frac{i}{8} \left[\mathcal{D}^{(g)}(x-z) \mathcal{D}^{(g)}(y-w) \delta^{ac} \delta^{bd} g_{\mu\rho} g_{\nu\sigma} + \mathcal{D}^{(g)}(x-w) \mathcal{D}^{(g)}(y-z) \delta^{ad} \delta^{bc} g_{\mu\sigma} g_{\nu\rho} \right], \qquad (3.58)
\end{aligned}$$

$$\begin{aligned}
K_I(x,y)^{ab}_{\mu\nu} &= -2 g_s f^{abc} \left[\partial^\mu G^{c\nu}(x) + G^{c\mu}(x) \partial_x^\nu - g^{\mu\nu} G^{c\sigma}(x) \partial_{x\sigma} \right] \delta^4(x-y) \\
&+ 2 i g_s^2 \left[\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-y) \gamma^\nu \frac{\lambda}{2}^b q_L(y) + (L \leftrightarrow R) \right] \\
&+ 2 g_s^3 \int d^4z \left[\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \not{G}(z) \mathcal{D}^{(L)}(z-y) \gamma^\nu \frac{\lambda}{2}^b q_L(y) + (L \leftrightarrow R) \right]
\end{aligned}$$

$$
\begin{aligned}
&- i g_s^4 \int d^4 z d^4 w \\
&\times \Bigg[\left(\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \frac{\lambda}{2}^c q_L(z) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_L(y) \gamma^\nu \frac{\lambda}{2}^b \mathcal{D}^{(L)}(y-w) \frac{\lambda}{2}^c q_L(w) \right) \\
&+ \left(\overline{q}_L(z) \frac{\lambda}{2}^c \mathcal{D}^{(L)}(z-x) \gamma^\mu \frac{\lambda}{2}^a q_L(x) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_L(w) \frac{\lambda}{2}^c \mathcal{D}^{(L)}(w-y) \gamma^\nu \frac{\lambda}{2}^b q_L(y) \right) \\
&- 2 \left(\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \frac{\lambda}{2}^c q_L(z) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_L(w) \frac{\lambda}{2}^c \mathcal{D}^{(L)}(w-y) \gamma^\nu \frac{\lambda}{2}^b q_L(y) \right) \\
&- \left(\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \frac{\lambda}{2}^c q_L(z) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_R(y) \gamma^\nu \frac{\lambda}{2}^b \mathcal{D}^{(R)}(y-w) \frac{\lambda}{2}^c q_R(w) \right) \\
&- \left(\overline{q}_L(z) \frac{\lambda}{2}^c \mathcal{D}^{(L)}(z-x) \gamma^\mu \frac{\lambda}{2}^a q_L(x) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_R(w) \frac{\lambda}{2}^c \mathcal{D}^{(R)}(w-y) \gamma^\nu \frac{\lambda}{2}^b q_R(y) \right) \\
&+ \left(\overline{q}_L(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(L)}(x-z) \frac{\lambda}{2}^c q_L(z) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_R(w) \frac{\lambda}{2}^c \mathcal{D}^{(R)}(w-y) \gamma^\nu \frac{\lambda}{2}^b q_R(y) \right) \\
&+ \left(\overline{q}_L(w) \frac{\lambda}{2}^c \mathcal{D}^{(L)}(w-y) \gamma^\nu \frac{\lambda}{2}^b q_L(y) \right) \mathcal{D}^{(s)}(z-w) \left(\overline{q}_R(x) \gamma^\mu \frac{\lambda}{2}^a \mathcal{D}^{(R)}(x-z) \frac{\lambda}{2}^c q_R(z) \right) \\
&+ (L \leftrightarrow R) \Bigg] ,
\end{aligned} \tag{3.59}
$$

respectively. Here $\partial_x^\nu, \partial_{x\sigma}$ in the first line of Eq.(3.59) means derivatives of KK gluons.

Dimension six operators induced from UED can be also obtained as taking KK particles are heavy enough than SM particles. We can show that $(K_I)^n$ with $n = 1, 2, 3$ once contribute $\mathcal{O}_{qqqq}$, respectively, and shown in Appendix C.2.1. Similarly, $(K_I)^n$ with $n = 1, 3$ once contribute $\mathcal{O}_{qqG}$, respectively, and $(K_I)^2$ twice contributes $\mathcal{O}_{qqG}$ and shown in Appendix C.2.2. As for $\mathcal{O}_{qqGG}$, K_I four times, $(K_I)^2$ six times, $(K_I)^3$ five times, and $(K_I)^4$ once contribute, respectively, and shown in Appendix C.2.3.

3.3 Little Higgs

In this subsection, we calculate dimension six operators with anomalous coupling in LH model. The Lagrangian is given by

$$
\mathcal{L}_{\text{int}} = -g_s \bar{t} \not{G} t - \frac{2}{3} e \bar{t} \not{A} t - \frac{g}{\sqrt{2}} \cos\beta (\bar{b} \not{W} P_L t + \text{h.c.}) - \frac{g}{\cos\theta_W} \bar{t} \not{Z} \left(-\frac{2}{3} \sin^2\theta_W + \frac{1}{2} \cos^2\beta P_L \right) t, \tag{3.60}
$$

where β and m_t are $\tan^{-1} \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2} \frac{v}{f}$ and $\frac{\sqrt{\lambda_1 \lambda_2}}{\lambda_1^2 + \lambda_2^2}$, respectively. The f denotes the VEV of the Little Higgs. When we take $\lambda_1 \simeq \lambda_2 \simeq 1$, we can estimate $\cos\beta \simeq 1 - \frac{v^2}{2f^2}$. Only 4-Fermi operators have non-standard effects originated from W, Z-boson exchanges in the electroweak interaction.

Then, below a energy scale of M_W, we can obtain $\mathcal{O}_{qqqq}$ as

$$\mathcal{O}^{\rm LH} = \frac{g^2}{2}\cos^2\beta\frac{1}{k^2-M_W^2}(\bar{t}\gamma^\mu P_L b)(\bar{b}\gamma^\mu P_L t) + \frac{g^2}{3}\tan^2\theta_W\cos^2\beta\frac{1}{k^2-M_Z^2}(\overline{q}\gamma^\mu P_L q)(\bar{t}\gamma^\mu P_L t). \tag{3.61}$$

They are useful for estimating an evidence of the LH.

4 Summary

It is important to obtain effective operators by integrating out high energy degrees of freedom in physics. In a quantum field theory, we can obtain effective Lagrangian by integrating out high energy momentum and heavy particles. In this paper, we have shown a general method of calculating accurate irrelevant effective operators in a scattering process without use of EOMs. By using this method, for example, we have represented coefficients of dimension six operators induced from SUSY, UED, and LH. We have shown coefficients of dimension six operators in QCD by integrating out sparticles (KK particles) in SUSY (UED). We have also shown coefficients of 4-Fermi operators originating from anomalous interactions in LH. They are promising candidates of beyond the SM, and our analyses will shed lights on the search of beyond the SM.

Our method can also give an effective action by integrating out high momentum degrees of freedom of massless particles. In this case, an effective Lagrangian have non-local interactions in general. Anyhow, our method can apply to other quantum field theories in any dimensions, so that we believe this technique is very useful in a lot of researches in physics.

Acknowledgments

This work is partially supported by Scientific Grant by Ministry of Education and Science, Nos. 20540272, 20039006, and 20025004.

A EOMs in a calculation of effective operators

We must be careful to deal with EOMs in a calculation of a general irrelevant operator, which is used for a scattering process. Let us consider a case that an effective operator is rewritten by a form of

$$S_{\rm eff}[\phi] = \int d^4x F[\phi]\frac{\delta\mathcal{L}[\phi]}{\delta\phi} \equiv F[\phi]\cdot\frac{\delta\mathcal{L}[\phi]}{\delta\phi}, \tag{A.62}$$

where $F[\phi]$ is a local functional containing ϕ and $\partial\phi$. Notice that Eq.(A.62) can be also obtained by a transformation of ϕ of

$$\phi(x) \longrightarrow \phi(x) + F[\phi(x)]. \tag{A.63}$$

The S-matrix should be unchanged under this transformation, which is so-called *equivalence theorem*[8, 9, 10]. Since $\frac{\delta\mathcal{L}[\phi]}{\delta\phi} = 0$ is regarded as EOM, we can always eliminate operator in Eq.(A.62) from effective theory by EOMs. In Green function's level, this operation is valid whenever all fields contained in Eq.(A.62) are on-shell[11]. However, if not all fields are on-shell, the Green function which contain the vertex Eq.(A.62) is not eliminated. We will show this situation as the following discussions.

At first, we show a case that all fields are on-shell. The Green function relation is given by

$$\begin{aligned} &\left\langle 0 \left| T\left[F[\phi]\frac{\delta\mathcal{L}[\phi]}{\delta\phi}(x)\phi(x_1)\cdots\phi(x_n)\right]\right| 0 \right\rangle \\ &= -\sum_i \delta(x-x_i)\left\langle 0\left| T\left[F[\phi(x_i)]\phi(x_1)\cdots\check{\phi}(x_i)\cdots\phi(x_n)\right]\right| 0\right\rangle, \end{aligned} \tag{A.64}$$

where the notation $\check{\phi}(x_i)$ means that $\phi(x_i)$ is excluded from right-hand side. This equation is so-called Schwinger-Dyson equation. We consider a scalar particle state, $\langle p|\phi(x)|0\rangle \neq 0$, and then the reduction formula gives

$$\begin{aligned} \left\langle p\left| F[\phi]\frac{\delta\mathcal{L}[\phi]}{\delta\phi}(x)\right| p'\right\rangle &= \int d^4z d^4z' e^{ipz}e^{-ip'z'}(p^2-m^2)(p'^2-m^2) \\ &\quad\times\left\langle 0\left| T\left[\phi(z)F[\phi]\frac{\delta\mathcal{L}[\phi]}{\delta\phi}(x)\phi(z')\right]\right| 0\right\rangle \\ &= -\int d^4z d^4z' e^{ipz}e^{-ip'z'}(p^2-m^2)(p'^2-m^2) \\ &\quad\times\left\{\delta(x-z)\left\langle 0\left|T\left[F[\phi](x)\phi(z')\right]\right|0\right\rangle + \delta(x-z')\left\langle 0\left|T\left[\phi(z)F[\phi](x)\right]\right|0\right\rangle\right\}. \end{aligned} \tag{A.65}$$

Here we use Eq.(A.64) in the second equality, and we can rewrite the last line by using a two point function, $f_\phi(x,z)$, which obviously has a pole at $p^2 = m^2$. Then, Eq.(A.65) can be written as

$$(\text{A.65}) = \int d^4z d^4z' e^{ipz}e^{-ip'z'}(p^2-m^2)(p'^2-m^2)\left[\delta(x-z)f_\phi(x,z') + \delta(x-z')f_\phi(z,x)\right]. \tag{A.66}$$

This means the matrix element should vanish at the on-shell.

Next, we show a case that all fields are not on-shell. The effective vertex becomes

$$F[\phi]\frac{\delta\mathcal{L}[\phi]}{\delta\phi}(x) \longrightarrow \overline{\psi}\Gamma^\mu\psi\frac{\delta\mathcal{L}}{\delta A^\mu}(x), \tag{A.67}$$

and we consider a process, $\psi_1\overline{\psi}_2 \to \psi_3\overline{\psi}_4$. Then, the S-matrix element,

$$\left\langle \psi_3\overline{\psi}_4\left| T\left[\overline{\psi}\Gamma^\mu\psi\frac{\delta\mathcal{L}}{\delta A^\mu}(x)\overline{\psi}\gamma^\nu\psi A_\nu(y)\right]\right|\psi_1\overline{\psi}_2\right\rangle,$$

is calculated as

$$\left\langle p_3p_4\left|T\left[\overline{\psi}\Gamma^\mu\psi\frac{\delta\mathcal{L}}{\delta A^\mu}(x)\ \overline{\psi}\gamma^\nu\psi A_\nu(y)\right]\right|p_1p_2\right\rangle$$
$$=\int d^4z_1d^4z_2d^4z_3d^4z_4\overline{V}(p_3;x_3)\overline{U}(p_4;x_4)(\not{p}_3-m_3)(\not{p}_4-m_4)$$
$$\times\left[\delta(x-y)f_\psi(z_3,x)f_\psi(z_4,x)f_\psi(z_1,y)f_\psi(z_2,y)\right](\not{p}_1-m_1)(\not{p}_2-m_2)U(p_2;x_2)V(p_1;x_1). \quad (A.68)$$

Here $f_\psi(z,x)$ denotes a fermion two point function which obviously has a pole at $\not{p}=m$, and U and V are wave functions for fermion and anti-fermion, respectively. Thus, we can find that there is a non-zero effect at on-shell for the process containing $F[\phi]\cdot\frac{\delta\mathcal{L}[\phi]}{\delta\phi}$ type effective operator. Consequently, when a scattering process has some contributions from such kind of operators, we can not eliminate these effective operators by EOMs in the effective Lagrangian.

Therefore, for the accurate estimations, we must be careful to deal with EOMs in a calculation of scattering amplitude. Hence, in this paper, we have shown the calculation method without EOMs and calculated some concrete effective operators.

B Fierz transformation and color factor

Here let us summarize some formulas which are useful for calculations. Fierz transformation of γ-matrix in Eq.(3.37) shows

$$(C^\dagger\gamma^\mu)_{ik}(\gamma^\nu C^T)_{lj}=-\frac{1}{2}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk}-\frac{1}{2}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \quad (B.69)$$
$$(C^\dagger)_{ik}(C^T)_{lj}=\frac{1}{4}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk}-\frac{1}{4}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}. \quad (B.70)$$

We should notice that scalar nor pseudo-scalar components do not appear by the Fierz transformation, since spinor components, (i,j,k,l), always have the same chirality in each set of (i,j) and (k,l).

The color factor becomes

$$\left(\frac{\lambda^b}{2}\right)_{jn}\left(\frac{\lambda^a}{2}\right)_{ni}\left(\frac{\lambda^b}{2}\right)_{lm}\left(\frac{\lambda^a}{2}\right)_{mk}=\frac{2}{9}\delta_{lk}\delta_{jk}-\frac{1}{3}\left(\frac{\lambda^a}{2}\right)_{lk}\left(\frac{\lambda^a}{2}\right)_{jk}, \quad (B.71)$$

from Fierz transformation of spinors. Next formulas are useful for the second term of Eq.(3.35), whose spinor and color factor are different from those of the first term. The spinor is given by

$$(\gamma^\mu)_{li}(\gamma_\mu)_{jk}=-\frac{1}{2}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk}+\frac{1}{2}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \quad (B.72)$$
$$\delta_{li}\delta_{jk}=\frac{1}{4}(\gamma^\mu)_{ji}(\gamma_\mu)_{lk}+\frac{1}{4}(\gamma^\mu\gamma^5)_{ji}(\gamma_\mu\gamma^5)_{lk}, \quad (B.73)$$

and the color factor becomes

$$\left(\frac{\lambda^b}{2}\right)_{jn}\left(\frac{\lambda^a}{2}\right)_{ni}\left(\frac{\lambda^a}{2}\right)_{lm}\left(\frac{\lambda^b}{2}\right)_{mk}=\frac{2}{9}\delta_{lk}\delta_{jk}+\frac{7}{6}\left(\frac{\lambda^a}{2}\right)_{lk}\left(\frac{\lambda^a}{2}\right)_{jk}, \quad (B.74)$$

while A,B are the same as the first term.

C Explicit Coefficients of dimension six operators

C.1 SUSY

In SUSY SM, dimension six operators are written as follows,

$$\mathcal{O}^{(1)}_{qqqq}(x) = \frac{12g_s^4}{192\pi^2}\left[C_{LL}\left(\bar{q}\gamma^\mu P_L q\right)\left(\bar{q}'\gamma_\mu P_L q'\right) + C_{RR}\left(\bar{q}\gamma^\mu P_R q\right)\left(\bar{q}'\gamma_\mu P_R q'\right)\right.$$
$$\left. + C_{LR}\left(\bar{q}\gamma^\mu P_L q\right)\left(\bar{q}'\gamma_\mu P_R q'\right) + C_{RL}\left(\bar{q}\gamma^\mu P_R q\right)\left(\bar{q}'\gamma_\mu P_L q'\right)\right], \tag{C.75}$$

$$\mathcal{O}^{(8)}_{qqqq}(x) = \frac{12g_s^4}{192\pi^2}\left[D_{LL}\left(\bar{q}T^a\gamma^\mu P_L q\right)\left(\bar{q}'T^a\gamma_\mu P_L q'\right) + D_{RR}\left(\bar{q}T^a\gamma^\mu P_R q\right)\left(\bar{q}'T^a\gamma_\mu P_R q'\right)\right.$$
$$\left. + D_{LR}\left(\bar{q}T^a\gamma^\mu P_L q\right)\left(\bar{q}'T^a\gamma_\mu P_R q'\right) + D_{RL}\left(\bar{q}T^a\gamma^\mu P_R q\right)\left(\bar{q}'T^a\gamma_\mu P_L q'\right)\right], \tag{C.76}$$

$$\mathcal{O}_{qqG}(x) = \frac{g_s^3}{96\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2+k_3)\overline{q}(k_2)T^a E^\mu_{L,R}G^a_\mu(k_3)P_{L,R}q(k_1), \tag{C.77}$$

$$\mathcal{O}_{qqGG}(x) = \frac{g_s^4}{192\pi^2}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2+k_3+k_4)$$
$$\overline{q}(k_1)\left[F^{\mu\nu}_{L,R}\delta^{ab} + H^{\mu\nu}_{L,R}T^aT^b\right]G^a_\mu(k_2)G^b_\nu(k_3)P_{L,R}q(k_4), \tag{C.78}$$

where $E^\mu_i,\ F^{\mu\nu}_i,\ H^{\mu\nu}_i,\ (i = L,\ R)$ are

$$E^\mu_i = \{e_{1i}\not{k}_1 + e_{2i}\not{k}_2\}k_1^\mu + \{e_{1i}\not{k}_2 + e_{2i}\not{k}_1\}k_2^\mu$$
$$+ \{e_{3i}(k_1^2+k_2^2) - e_{4i}k_1\cdot k_2\}\gamma^\mu - e_{5i}i\epsilon^{\alpha\beta\mu\nu}\gamma_5\gamma_\nu k_{1\alpha}k_{2\beta}, \tag{C.79}$$

$$F^{\mu\nu}_i = f_{1i\alpha}i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2i\alpha}g^{\mu\nu}\gamma^\alpha + f_{3i\alpha}g^{\alpha\mu}\gamma^\nu + f_{4i\alpha}g^{\alpha\nu}\gamma^\mu, \tag{C.80}$$

$$H^{\mu\nu}_i = h_{1i\alpha}i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2i\alpha}g^{\mu\nu}\gamma^\alpha + h_{3i\alpha}g^{\alpha\mu}\gamma^\nu + h_{4i\alpha}g^{\alpha\nu}\gamma^\mu. \tag{C.81}$$

C.1.1 Coefficients in $\mathcal{O}_{qqqq}$

The coefficients of 4-Fermi operator are given as

$$C^{\rm SUSY}_{LL} = \frac{2}{9}[f_1(m_{\tilde{q}_L}, m_{\tilde{q}'_L}) + f_2(m_{\tilde{q}_L}, m_{\tilde{q}'_L})], \tag{C.82}$$

$$C^{\rm SUSY}_{RR} = \frac{2}{9}[f_1(m_{\tilde{q}_R}, m_{\tilde{q}'_R}) + f_2(m_{\tilde{q}_R}, m_{\tilde{q}'_R})], \tag{C.83}$$

$$C^{\rm SUSY}_{LR} = -\frac{2}{9}[f_1(m_{\tilde{q}_R}, m_{\tilde{q}'_L}) - f_2(m_{\tilde{q}_L}, m_{\tilde{q}'_R})], \tag{C.84}$$

$$C^{\rm SUSY}_{RL} = -\frac{2}{9}[f_1(m_{\tilde{q}_L}, m_{\tilde{q}'_R}) + f_2(m_{\tilde{q}_L}, m_{\tilde{q}'_R})], \tag{C.85}$$

$$D^{\rm SUSY}_{LL} = -\frac{1}{3}f_1(m_{\tilde{q}_L}, m_{\tilde{q}'_L}) - \frac{7}{6}f_2(m_{\tilde{q}_L}, m_{\tilde{q}'_L}), \tag{C.86}$$

$$D^{\rm SUSY}_{RR} = -\frac{1}{3}f_1(m_{\tilde{q}_R}, m_{\tilde{q}'_R}) - \frac{7}{6}f_2(m_{\tilde{q}_R}, m_{\tilde{q}'_R}), \tag{C.87}$$

$$D_{LR}^{\rm SUSY} = -\frac{7}{6} f_1(m_{\tilde{q}_L}, m_{\tilde{q}'_R}) - \frac{1}{3} f_2(m_{\tilde{q}_L}, m_{\tilde{q}'_R}), \tag{C.88}$$
$$D_{RL}^{\rm SUSY} = -\frac{7}{6} f_2(m_{\tilde{q}_R}, m_{\tilde{q}'_L}) - \frac{1}{3} f_1(m_{\tilde{q}_R}, m_{\tilde{q}'_L}), \tag{C.89}$$

$$f_1(m_{\tilde{q}}, m_{\tilde{q}'}) = \int_0^1 dy \int_0^1 dz \frac{yz^2}{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)yz + (m_{\tilde{q}}^2 - m_{\tilde{q}'}^2)z + m_{\tilde{q}'}}, \tag{C.90}$$
$$f_2(m_{\tilde{q}}, m_{\tilde{q}'}) = \int_0^1 dy \int_0^1 dz \frac{m_{\tilde{g}}^2 yz^2}{[(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)yz + (m_{\tilde{q}}^2 - m_{\tilde{q}'}^2)z + m_{\tilde{q}'}]^2}. \tag{C.91}$$

C.1.2 Coefficients in $\mathcal{O}_{qqG}$

The coefficients of q-q-G operator are given as

$$E_L^\mu \equiv E^\mu(m_{\tilde{q}} = m_{\tilde{q}_L}), \tag{C.92}$$
$$= \{e_1(m_{\tilde{q}_L})\not{k}_1 + e_2(m_{\tilde{q}_L})\not{k}_2\}k_1^\mu + \{e_1(m_{\tilde{q}_L})\not{k}_2 + e_2(m_{\tilde{q}_L})\not{k}_1\}k_2^\mu$$
$$+ \{e_3(m_{\tilde{q}_L})(k_1^2 + k_2^2) - e_4(m_{\tilde{q}_L})k_1 \cdot k_2\}\gamma^\mu - e_5(m_{\tilde{q}_L}) i\epsilon^{\alpha\beta\mu\nu}\gamma_5\gamma_\nu k_{1\alpha}k_{2\beta}, \tag{C.93}$$
$$E_R^\mu = E^\mu(m_{\tilde{q}_R}), \tag{C.94}$$

$$e_1(m_{\tilde{q}})$$
$$= \frac{107m_{\tilde{g}}^6 - 495m_{\tilde{g}}^4 m_{\tilde{q}}^2 + 477m_{\tilde{g}}^2 m_{\tilde{q}}^4 - 89m_{\tilde{q}}^6 - 6(m_{\tilde{g}}^6 + 3m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{18(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{C.95}$$

$$e_2(m_{\tilde{q}})$$
$$= \frac{-203m_{\tilde{g}}^6 + 351m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 189m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 41m_{\tilde{q}}^6 + 6(m_{\tilde{g}}^6 + 51m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{18(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{C.96}$$

$$e_3(m_{\tilde{q}}) = e_2(m_{\tilde{q}}), \tag{C.97}$$

$$e_4(m_{\tilde{q}})$$
$$= \frac{-155m_{\tilde{g}}^6 + 423m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 333m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 65m_{\tilde{q}}^6 + 6(m_{\tilde{g}}^6 + 27m_{\tilde{g}}^4 m_{\tilde{q}}^2 - 54m_{\tilde{g}}^2 m_{\tilde{q}}^4 + 18m_{\tilde{q}}^6)\log(m_{\tilde{g}}^2/m_{\tilde{q}}^2)}{9(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^4}, \tag{C.98}$$

$$e_5(m_{\tilde{q}}) = \frac{9(m_{\tilde{g}}^4 - m_{\tilde{q}}^4 - 2m_{\tilde{g}}^2 m_{\tilde{q}}^2 \log(m_{\tilde{g}}^2/m_{\tilde{q}}^2))}{(m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^3}. \tag{C.99}$$

C.1.3 Coefficients in $\mathcal{O}_{qqGG}$

The coefficients of q-q-G-G operator are given as

$$F^{\mu\nu} = f_{1\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2\alpha} g^{\mu\nu}\gamma^\alpha + f_{3\alpha} g^{\alpha\mu}\gamma^\nu + f_{4\alpha} g^{\alpha\nu}\gamma^\mu, \tag{C.100}$$

$$H^{\mu\nu} = h_{1\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2\alpha} g^{\mu\nu}\gamma^\alpha + h_{3\alpha} g^{\alpha\mu}\gamma^\nu + h_{4\alpha} g^{\alpha\nu}\gamma^\mu, \tag{C.101}$$

$$f_{1\alpha} = \left[\frac{1}{2}S_{0\alpha} - 2(P_0 + Q_0)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.102}$$

$$f_{2\alpha} = \left[-3K_\alpha + \frac{1}{2}R_{3\alpha} + \frac{1}{2}S_{1\alpha} + 2(P_1 + Q_1)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.103}$$

$$f_{3\alpha} = \left[\frac{1}{2}R_{2\alpha} + \frac{1}{2}S_{2\alpha} + 2(P_3 + Q_3)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.104}$$

$$f_{4\alpha} = \left[\frac{1}{2}R_{1\alpha} + \frac{1}{2}S_{3\alpha} + 2(P_2 + Q_2)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.105}$$

$$h_{1\alpha} = [12(P_0 + Q_0)_\alpha](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.106}$$

$$h_{2\alpha} = \left[2K_\alpha - \frac{1}{3}R_{3\alpha} + 12(P_1 + Q_1)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.107}$$

$$h_{3\alpha} = \left[-\frac{1}{3}R_{2\alpha} + 12(P_3 + Q_3)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}), \tag{C.108}$$

$$h_{4\alpha} = \left[-\frac{1}{3}R_{1\alpha} + 12(P_2 + Q_2)_\alpha\right](m_a = m_{\tilde{g}}, m_b = m_{\tilde{q}_i}).,\tag{C.109}$$

where $K(m_a, m_b), P(m_a, m_b), Q(m_a, m_b), R(m_a, m_b), S(m_a, m_b)$ are written in terms of a linear combination of the momentums;

$$K_\alpha = a_1 k_{1\alpha} + a_2 k_{4\alpha}, \tag{C.110}$$

$$P_{0\alpha} = (i_{11} + i_{12} + i_{13})k_{1\alpha} + (i_{21} + i_{22} + i_{23})k_{2\alpha} + (i_{31} + i_{32} + i_{33})k_{4\alpha}, \tag{C.111}$$

$$P_{1\alpha} = (i_{11} + i_{12} - i_{13})k_{1\alpha} + (i_{21} + i_{22} - i_{23})k_{2\alpha} + (i_{31} + i_{32} - i_{33})k_{4\alpha}, \tag{C.112}$$

$$P_{2\alpha} = (i_{11} - i_{12} + i_{13})k_{1\alpha} + (i_{21} - i_{22} + i_{23})k_{2\alpha} + (i_{31} - i_{32} + i_{33})k_{4\alpha}, \tag{C.113}$$

$$P_{3\alpha} = (-i_{11} + i_{12} + i_{13})k_{1\alpha} + (-i_{21} + i_{22} + i_{23})k_{2\alpha} + (-i_{31} + i_{32} + i_{33})k_{4\alpha}, \tag{C.114}$$

$$Q_{0\alpha} = (j_{11} - j_{12} + j_{13})k_{1\alpha} + (j_{21} - j_{22} + j_{23})k_{2\alpha} + (j_{31} - j_{32} + j_{33})k_{4\alpha}, \tag{C.115}$$

$$Q_{1\alpha} = Q_{0\alpha}, \tag{C.116}$$

$$Q_{2\alpha} = (-j_{11} + j_{12} + j_{13})k_{1\alpha} + (-j_{21} + j_{22} + j_{23})k_{2\alpha} + (-j_{31} + j_{32} + j_{33})k_{4\alpha}, \tag{C.117}$$

$$Q_{3\alpha} = (j_{11} + j_{12} - j_{13})k_{1\alpha} + (j_{21} + j_{22} - j_{23})k_{2\alpha} + (j_{31} + j_{32} - j_{33})k_{4\alpha}, \tag{C.118}$$

$$R_{1\alpha} = b_{11}k_{1\alpha} + b_{21}k_{2\alpha} + b_{31}k_{4\alpha}, \tag{C.119}$$

$$R_{2\alpha} = b_{12}k_{1\alpha} + b_{22}k_{2\alpha} + b_{32}k_{4\alpha}, \tag{C.120}$$

$$R_{3\alpha} = b_{13}k_{1\alpha} + b_{23}k_{2\alpha} + b_{33}k_{4\alpha}, \tag{C.121}$$

$$S_{0\alpha} = (-f_{12} + f_{13})k_{1\alpha} + (-f_{22} + f_{23})k_{2\alpha} + (-f_{32} + f_{33})k_{4\alpha}, \tag{C.122}$$

$$S_{1\alpha} = (f_{12} + f_{13})k_{1\alpha} + (f_{22} + f_{23})k_{2\alpha} + (f_{32} + f_{33})k_{4\alpha}, \tag{C.123}$$

$$S_{2\alpha} = S_{1\alpha}, \tag{C.124}$$

$$\begin{aligned} S_{3\alpha} = {} & (h_1 - 2f_{11} - f_{12} - f_{13})k_{1\alpha} + (h_2 - 2f_{21} - f_{22} - f_{23})k_{2\alpha} \\ & + (h_3 - 2f_{31} - f_{32} - f_{33})k_{4\alpha}. \end{aligned} \tag{C.125}$$

These coefficients, $a_1, a_2, \cdots$, are given in terms of Feynman parameter integral as

$$a_1(m_a, m_b) = \int_0^1 dx \int_0^1 dy \frac{2xy^2 - 2y^2}{xy\,(m_a^2 - m_b^2) + m_b^2}, \tag{C.126}$$

$$a_2(m_a, m_b) = \int_0^1 dx \int_0^1 dy \frac{2y^2 - 2y}{xy\,(m_a^2 - m_b^2) + m_b^2}, \tag{C.127}$$

$$i_{11}(m_a, m_b) = -\int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6y^2z^3}{z\,(m_a^2 - m_b^2) + m_b^2}, \tag{C.128}$$

$$\begin{aligned} i_{12}(m_a, m_b) &= i_{13}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(1 - yz)}{z\,(m_a^2 - m_b^2) + m_b^2}, \end{aligned} \tag{C.129}$$

$$\begin{aligned} i_{21}(m_a, m_b) &= i_{22}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(yz - xyz)}{z\,(m_a^2 - m_b^2) + m_b^2}, \end{aligned} \tag{C.130}$$

$$i_{23}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-xyz + yz - 1)}{z\,(m_a^2 - m_b^2) + m_b^2}, \tag{C.131}$$

$$i_{31}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(yz - z + 1)}{z\,(m_a^2 - m_b^2) + m_b^2}, \tag{C.132}$$

$$\begin{aligned} i_{32}(m_a, m_b) &= i_{33}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(yz - z)}{z\,(m_a^2 - m_b^2) + m_b^2}, \end{aligned} \tag{C.133}$$

$$\begin{aligned} j_{11}(m_a, m_b) &= j_{12}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_a^2(1 - yz)}{(z\,(m_a^2 - m_b^2) + m_b^2)^2} \end{aligned} \tag{C.134}$$

$$j_{13}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{6y^2z^3m_a^2}{(z\,(m_a^2 - m_b^2) + m_b^2)^2}, \tag{C.135}$$

$$\begin{aligned} j_{21}(m_a, m_b) &= j_{23}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_a^2(yz - xyz)}{(z\,(m_a^2 - m_b^2) + m_b^2)^2}, \end{aligned} \tag{C.136}$$

$$j_{22}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a^2(-xyz + yz - 1)}{\left(z\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.137}$$

$$\begin{aligned} j_{31}(m_a, m_b) &= j_{32}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a^2(yz - z)}{\left(z\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \end{aligned} \tag{C.138}$$

$$j_{33}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a^2(yz - z + 1)}{\left(z\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.139}$$

$$b_{11}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(4xyz - 4z + 2)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.140}$$

$$b_{12}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24yz^2(xyz - z + 1)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.141}$$

$$b_{13}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(4xyz - 4z)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.142}$$

$$b_{21}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4yz + 4z - 2)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.143}$$

$$b_{22}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4yz + 4z - 2)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.144}$$

$$b_{23}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24yz^2(z - yz)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.145}$$

$$b_{31}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24yz^2(z - yz)}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.146}$$

$$\begin{aligned} b_{32}(m_a, m_b) &= b_{33}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24y(z - 1)z^2}{xyz\left(m_a^2 - m_b^2\right) + m_b^2}, \end{aligned} \tag{C.147}$$

$$f_{11}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(2yz - 2z + 1)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.148}$$

$$\begin{aligned} f_{12}(m_a, m_b) &= f_{13}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2(yz - z)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \end{aligned} \tag{C.149}$$

$$f_{21}(m_a, m_b) = 0 \tag{C.150}$$

$$f_{22}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2(yz - z)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.151}$$

$$f_{23}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2(xyz - yz + z - 1)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.152}$$

$$f_{31}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(2z-1)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.153}$$

$$\begin{aligned} f_{32}(m_a, m_b) &= f_{33}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12y(z-1)z^2}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \end{aligned} \tag{C.154}$$

$$h_1(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a^2(2yz - 2z + 1)}{\left(yz\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.155}$$

$$h_2(m_a, m_b) = 0, \tag{C.156}$$

$$h_3(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(2z-1)m_a^2}{\left(yz\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}. \tag{C.157}$$

C.2 UED

Let us list the dimension six operators of QCD in UED.

C.2.1 Coefficients in $\mathcal{O}_{qqqq}$

When $(\overline{q}q)(\overline{q}'q')$ chirality is (LL)(LL) or (RR)(RR), 4-Fermi operator is

$$\mathcal{O}_{qqqq}(x) = \frac{g_s^4}{192\pi^2}\left[f_1(\overline{q}\gamma^\mu q)(\overline{q}'\gamma_\mu q') + f_2\left(\overline{q}\gamma^\mu \frac{\lambda}{2}^a q\right)\left(\overline{q}'\gamma^\mu \frac{\lambda}{2}^a q'\right)\right], \tag{C.158}$$

and the coefficients f_1, f_2 are

$$f_{1L} = \frac{4\left(m_g^4 + 4m_g^2 m_L^2 \log\left(\frac{m_L}{m_g}\right) - m_L^4\right)}{\left(m_g^2 - m_L^2\right)^3}, \tag{C.159}$$

$$f_{1R} = \frac{4\left(m_g^4 + 4m_g^2 m_R^2 \log\left(\frac{m_R}{m_g}\right) - m_R^4\right)}{\left(m_g^2 - m_R^2\right)^3}, \tag{C.160}$$

$$\begin{aligned} f_{2L} = &\frac{1}{\left(m_g^2 - m_5^2\right)(m_5 - m_L)^2(m_5 + m_L)^2(m_g - m_L)^2(m_g + m_L)^2} \\ &\left[18m_L^2\left(m_g^2\left(m_5^4 \log\left(\frac{m_L^2}{m_g^2}\right) - m_L^4 \log\left(\frac{m_g^2}{m_L^2}\right)\right) + m_5^2\left(m_g^4 + m_L^4\right)\log\left(\frac{m_5^2}{m_L^2}\right)\right.\right. \\ &+ 2m_5^2 m_g^2 m_L^2 \log\left(\frac{m_g^2}{m_5^2}\right) \\ &\left. + (m_5^2 - m_g^2)(m_5^2 - m_L^2)(m_g^2 - m_L^2)\right) + \frac{7\left(-m_5^4 + 2m_5^2 m_L^2 \log\left(\frac{m_5^2}{m_L^2}\right) + m_L^4\right)}{2\left(m_5^2 - m_L^2\right)^3} \\ &\left. + \frac{30\left(-m_g^4 + 2m_g^2 m_L^2 \log\left(\frac{m_g^2}{m_L^2}\right) + m_L^4\right)}{\left(m_g^2 - m_L^2\right)^3}\right], \end{aligned} \tag{C.161}$$

$$
\begin{aligned}
f_{2R} = & \frac{1}{\left(m_g^2 - m_5^2\right)(m_5 - m_R)^2(m_5 + m_R)^2(m_g - m_R)^2(m_g + m_R)^2} \\
& \left[18m_R^2 \left(m_g^2 \left(m_5^4 \log\left(\frac{m_R^2}{m_g^2}\right) - m_R^4 \log\left(\frac{m_g^2}{m_R^2}\right)\right) + m_5^2\left(m_g^4 + m_R^4\right)\log\left(\frac{m_5^2}{m_R^2}\right)\right.\right. \\
& + 2m_5^2 m_g^2 m_R^2 \log\left(\frac{m_g^2}{m_5^2}\right) \\
& \left. +(m_5^2 - m_g^2)(m_5^2 - m_R^2)(m_g^2 - m_R^2)\right) + \frac{7\left(-m_5^4 + 2m_5^2 m_R^2 \log\left(\frac{m_5^2}{m_R^2}\right) + m_R^4\right)}{2\left(m_5^2 - m_R^2\right)^3} \\
& \left. + \frac{30\left(-m_g^4 + 2m_g^2 m_R^2 \log\left(\frac{m_g^2}{m_R^2}\right) + m_R^4\right)}{\left(m_g^2 - m_R^2\right)^3}\right].
\end{aligned}
\tag{C.162}
$$

When $(\overline{u}u)(\bar{t}t)$ chirality is (LL)(RR) or (RR)(LL), 4-Fermi operator is

$$
\mathcal{O}_{qqqq}(x) = \frac{g_s^4}{192\pi^2}\left[f_3(\overline{q}\gamma^\mu q)(\overline{q}'\gamma_\mu q') + f_4\left(\overline{q}\gamma^\mu \frac{\lambda}{2}^a q\right)\left(\overline{q}'\gamma^\mu \frac{\lambda}{2}^a q'\right)\right],
\tag{C.163}
$$

and the coefficients f_3, f_4 are

$$
\begin{aligned}
f_3(m_q, m_{q'}) = & -\frac{1}{\left(m_g^2 - m_q^2\right)^2\left(m_g^2 - m_{q'}^2\right)^2\left(m_q^2 - m_{q'}^2\right)} \\
& \left[8\left(2m_g^2 m_{q'}^4\left(m_g^2 - 2m_q^2\right)\log(m_g) - 2m_{q'}^4\left(m_g^2 - m_q^2\right)^2\log(m_{q'})\right.\right. \\
& + m_g^2(m_g^2 - m_q^2)(m_g^2 - m_{q'}^2)(m_q^2 - m_{q'}^2) \\
& \left.\left. +2m_q^4\left(m_g^4\log\left(\frac{m_q}{m_g}\right) + 2m_g^2 m_{q'}^2\log\left(\frac{m_g}{m_q}\right) + m_{q'}^4\log(m_q)\right)\right)\right],
\end{aligned}
\tag{C.164}
$$

$$
\begin{aligned}
f_4(m_q, m_{q'}) = & \frac{9}{2\left(m_5^2 - m_q^2\right)\left(m_5^2 - m_{q'}^2\right)\left(m_q^2 - m_{q'}^2\right)} \\
& \left(\frac{\left(m_5^2 - m_q^2\right)\left(-m_g^4 + 4m_g^4\log(m_g) + m_{q'}^4 - 4m_{q'}^4\log(m_{q'})\right)}{m_g^2 - m_{q'}^2}\right. \\
& - \frac{\left(m_5^2 - m_{q'}^2\right)\left(-m_g^4 + 4m_g^4\log(m_g) + m_q^4 - 4m_q^4\log(m_q)\right)}{m_g^2 - m_q^2} \\
& \left. + \frac{\left(m_q^2 - m_{q'}^2\right)\left(-m_5^4 + 4m_5^4\log(m_5) + m_g^4 - 4m_g^4\log(m_g)\right)}{m_5^2 - m_g^2}\right) \\
& + \frac{9}{2\left(m_5^2 - m_q^2\right)^2\left(m_5^2 - m_{q'}^2\right)^2\left(m_{q'}^2 - m_q^2\right)} \\
& \left[2m_5^2 m_{q'}^4\left(m_5^2 - 2m_q^2\right)\log(m_5) - 2m_{q'}^4\left(m_5^2 - m_q^2\right)^2\log(m_{q'})\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ m_5^2(m_5^2 - m_q^2)(m_5^2 - m_{q'}^2)(m_q^2 - m_{q'}^2) \\
&+2m_q^4 \left(m_5^4 \log\left(\frac{m_q}{m_5}\right) + 2m_5^2 m_{q'}^2 \log\left(\frac{m_5}{m_q}\right) + m_{q'}^4 \log(m_q) \right) \Bigg] \\
&- \frac{30}{\left(m_g^2 - m_q^2\right)^2 \left(m_g^2 - m_{q'}^2\right)^2 \left(m_q^2 - m_{q'}^2\right)} \\
&\Big[2m_g^2 m_{q'}^4 \left(m_g^2 - 2m_q^2\right) \log(m_g) - 2m_{q'}^4 \left(m_g^2 - m_q^2\right)^2 \log(m_{q'}) \\
&+ m_g^2(m_g^2 - m_q^2)(m_g^2 - m_{q'}^2)(m_q^2 - m_{q'}^2) \\
&+2m_q^4 \left(m_g^4 \log\left(\frac{m_q}{m_g}\right) + 2m_g^2 m_{q'}^2 \log\left(\frac{m_g}{m_q}\right) + m_{q'}^4 \log(m_q) \right) \Bigg], \qquad \text{(C.165)}
\end{aligned}
$$

where $m_q, m_{q'}$ are m_L or m_R $(m_q, \neq m_{q'})$.

C.2.2 Coefficients in $\mathcal{O}_{qqG}$

q-q-G operator in UED takes the form as

$$
\mathcal{O}_{qqG} = -\frac{g_s^3}{192\pi^2} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^4(-k_1 + k_2 + k_3) \overline{q}(k_1) C^\mu G_\mu(k_3) q(k_2), \qquad \text{(C.166)}
$$

$$
C^\mu \equiv c_1 i\epsilon^{\alpha\beta\mu\nu}\gamma^5\gamma_\nu + c_2 g^{\mu\alpha}\gamma^\beta + c_3 g^{\mu\beta}\gamma^\alpha + c_4 g^{\alpha\beta}\gamma^\mu, \qquad \text{(C.167)}
$$

$$
c_1 \equiv c_{11} k_{1\alpha} k_{2\beta}, \qquad \text{(C.168)}
$$

$$
c_2 \equiv c_{21} k_{1\alpha} k_{1\beta} + c_{22} k_{1\alpha} k_{2\beta} + c_{23} k_{2\alpha} k_{1\beta} + c_{24} k_{2\alpha} k_{2\beta}, \qquad \text{(C.169)}
$$

$$
c_3 \equiv c_{31} k_{1\alpha} k_{1\beta} + c_{32} k_{1\alpha} k_{2\beta} + c_{33} k_{2\alpha} k_{1\beta} + c_{34} k_{2\alpha} k_{2\beta}, \qquad \text{(C.170)}
$$

$$
c_4 \equiv c_{41} k_{1\alpha} k_{1\beta} + c_{42} k_{1\alpha} k_{2\beta} + c_{43} k_{2\alpha} k_{2\beta}. \qquad \text{(C.171)}
$$

$$
\qquad \text{(C.172)}
$$

Note that only c_4 takes the different form between the quark chirality q_L and q_R, because q-$q^{(n)}$-$G_5^{(n)}$ vertex is chiral interaction. The coefficients are given as

$$
c_{11} = 6\frac{m_g^4 - 4m_g^2 m_q^2 \log\left(\frac{m_g}{m_q}\right) - m_q^4}{\left(m_g^2 - m_q^2\right)^3}, \qquad \text{(C.173)}
$$

$$
c_{21} = 6\frac{5m_g^6 - 27m_g^4 m_q^2 + 27m_g^2 m_q^4 + 12m_g^4\left(m_g^2 - 3m_q^2\right)\log\left(\frac{m_q}{m_g}\right) - 5m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \qquad \text{(C.174)}
$$

$$
c_{22} = 6\frac{7m_g^6 - 27m_g^2 m_q^4 + 12\left(m_g^6 - 6m_g^4 m_q^2 + 6m_g^2 m_q^4\right)\log\left(\frac{m_g}{m_q}\right) + 20m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \qquad \text{(C.175)}
$$

$$
c_{23} = 6\frac{12m_g^6 \log\left(\frac{m_g}{m_q}\right) - 11m_g^6 + 18m_g^4 m_q^2 - 9m_g^2 m_q^4 + 2m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \qquad \text{(C.176)}
$$

$$c_{24} = 6\frac{12m_g^6\log\left(\frac{m_q}{m_g}\right) + 11m_g^6 - 18m_g^4m_q^2 + 9m_g^2m_q^4 - 2m_q^6}{18\left(m_g^2 - m_q^2\right)^4}, \tag{C.177}$$

$$c_{31} = \frac{1}{3}\left(\frac{5m_g^6 - 27m_g^4m_q^2 + 27m_g^2m_q^4 + 12\left(m_g^6 - 3m_g^4m_q^2\right)\log\left(\frac{m_q}{m_g}\right) - 5m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$
$$\left.-\frac{6\left(2m_5^6 - 9m_5^4m_q^2 + 18m_5^2m_q^4 + 12m_q^6\log\left(\frac{m_q}{m_5}\right) - 11m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right), \tag{C.178}$$

$$c_{32} = \frac{1}{3}\left(\frac{3\left(7m_5^6 - 36m_5^4m_q^2 + 45m_5^2m_q^4 + 12\left(3m_5^2m_q^4 - 2m_q^6\right)\log\left(\frac{m_5}{m_q}\right) - 16m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right.$$

$$\left.+\frac{7m_g^6 - 27m_g^2m_q^4 + 12\left(m_g^6 - 6m_g^4m_q^2 + 6m_g^2m_q^4\right)\log\left(\frac{m_g}{m_q}\right) + 20m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right), \tag{C.179}$$

$$c_{33} = \frac{1}{3}\left(\frac{12m_g^6\log\left(\frac{m_g}{m_q}\right) - 11m_g^6 + 18m_g^4m_q^2 - 9m_g^2m_q^4 + 2m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$
$$\left.-\frac{3\left(2m_5^6 - 9m_5^4m_q^2 + 18m_5^2m_q^4 + 12m_q^6\log\left(\frac{m_q}{m_5}\right) - 11m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right), \tag{C.180}$$

$$c_{34} = \frac{1}{3}\left(\frac{12m_g^6\log\left(\frac{m_q}{m_g}\right) + 11m_g^6 - 18m_g^4m_q^2 + 9m_g^2m_q^4 - 2m_q^6}{\left(m_g^2 - m_q^2\right)^4}\right.$$

$$\left.-\frac{3\left(-5m_5^6 + 27m_5^4m_q^2 - 27m_5^2m_q^4 + 12m_q^4\left(m_q^2 - 3m_5^2\right)\log\left(\frac{m_5}{m_q}\right) + 5m_q^6\right)}{\left(m_5^2 - m_q^2\right)^4}\right), \tag{C.181}$$

$$c_{41L} = \frac{18m_gm_L}{\left(m_g^2 - m_5^2\right)^3(m_5 - m_L)(m_5 + m_L)(m_g - m_L)^3(m_g + m_L)^3}$$
$$\Big[4m_g^2m_L^2\left(m_5^2 - m_g^2\right)^3\log(m_L) + 4m_5^2m_g^2\log(m_5)\left(m_g^2 - m_L^2\right)^3$$
$$+\left(m_5^2 - m_L^2\right)\left((m_5^2 - m_g^2)(m_g^2 - m_L^2)\left(m_5^2\left(m_g^2 + m_L^2\right) - 3m_g^4 + m_g^2m_L^2\right)\right.$$
$$\left.-4m_g^2\log(m_g)\left(-3m_5^2m_g^2m_L^2 + m_5^2m_L^2\left(m_5^2 + m_L^2\right) + m_g^6\right)\right)\Big]$$

$$+\frac{5m_5^6-27m_5^4m_L^2+27m_5^2m_L^4-12m_L^4\left(m_L^2-3m_5^2\right)\log\left(\frac{m_5}{m_L}\right)-5m_L^6}{2\left(m_5^2-m_L^2\right)^4}$$
$$+\frac{5\left(-5m_g^6+27m_g^4m_L^2-27m_g^2m_L^4+12\left(m_g^6-3m_g^4m_L^2\right)\log\left(\frac{m_g}{m_L}\right)+5m_L^6\right)}{3\left(m_g^2-m_L^2\right)^4}$$
$$-\frac{5m_g^6-27m_g^4m_L^2+27m_g^2m_L^4+12\left(m_g^6-3m_g^4m_L^2\right)\log\left(\frac{m_L}{m_g}\right)-5m_L^6}{3\left(m_g^2-m_L^2\right)^4}, \tag{C.182}$$

$$c_{42L}=\frac{1}{3}\left(-\frac{108m_gm_L}{\left(m_g^2-m_5^2\right)^3(m_5-m_L)^2(m_5+m_L)^2(m_g-m_L)^2(m_g+m_L)^2}\right.$$
$$\left[-2m_L^4\left(m_5^2-m_g^2\right)^3\log(m_L)+\left(m_L^2-m_5^2\right)\left((m_5^2-m_g^2)(m_g^2-m_L^2)\left(2m_5^2m_g^2-m_L^2\left(m_5^2+m_g^2\right)\right)\right.\right.$$
$$\left.+2m_g^2(m_5-m_L)(m_5+m_L)\log(m_g)\left(m_5^2\left(m_g^2-2m_L^2\right)+m_g^4\right)\right)$$
$$\left.+2m_5^2\log(m_5)\left(m_g^2-m_L^2\right)^2\left(m_5^4+m_5^2m_g^2-2m_g^2m_L^2\right)\right]$$

$$+\frac{3\left(-2m_5^6+9m_5^4m_L^2-18m_5^2m_L^4+12m_L^6\log\left(\frac{m_5}{m_L}\right)+11m_L^6\right)}{\left(m_5^2-m_L^2\right)^4}$$
$$+\frac{-12m_g^6\log\left(\frac{m_g}{m_L}\right)+11m_g^6-18m_g^4m_L^2+9m_g^2m_L^4-2m_L^6}{\left(m_g^2-m_L^2\right)^4}$$
$$-\frac{7m_g^6-27m_g^2m_L^4+12\left(m_g^6-6m_g^4m_L^2+6m_g^2m_L^4\right)\log\left(\frac{m_g}{m_L}\right)+20m_L^6}{\left(m_g^2-m_L^2\right)^4}$$
$$\left.+\frac{10\left(12m_g^6\log\left(\frac{m_L}{m_g}\right)+11m_g^6-18m_g^4m_L^2+9m_g^2m_L^4-2m_L^6\right)}{\left(m_g^2-m_L^2\right)^4}\right), \tag{C.183}$$

$$c_{43L}=-\frac{18m_gm_L}{\left(m_g^2-m_5^2\right)^3(m_5-m_L)^3(m_5+m_L)^3(m_g-m_L)(m_g+m_L)}$$
$$\left(4m_5^2\left(m_L^2\left(m_g^2-m_5^2\right)^3\log(m_L)+m_g^2\left(m_5^2-m_L^2\right)^3\log(m_g)\right.\right.$$
$$\left.+\log(m_5)(m_L^2-m_g^2)\left(m_5^6-3m_5^2m_g^2m_L^2+m_g^2m_L^2\left(m_g^2+m_L^2\right)\right)\right)$$
$$\left.+(m_5^2-m_g^2)(m_5^2-m_L^2)(m_g^2-m_L^2)\left(3m_5^4-m_5^2\left(m_g^2+m_L^2\right)-m_g^2m_L^2\right)\right)$$
$$+\frac{5m_5^6-27m_5^4m_L^2+27m_5^2m_L^4-12m_L^4\left(m_L^2-3m_5^2\right)\log\left(\frac{m_5}{m_L}\right)-5m_L^6}{2\left(m_5^2-m_L^2\right)^4}$$

$$+\frac{5\left(-5m_g^6+27m_g^4m_L^2-27m_g^2m_L^4+12\left(m_g^6-3m_g^4m_L^2\right)\log\left(\frac{m_g}{m_L}\right)+5m_L^6\right)}{3\left(m_g^2-m_L^2\right)^4}$$

$$+\frac{-12m_g^6\log\left(\frac{m_L}{m_g}\right)-11m_g^6+18m_g^4m_L^2-9m_g^2m_L^4+2m_L^6}{3\left(m_g^2-m_L^2\right)^4}, \tag{C.184}$$

$$c_{41R}=\frac{18m_gm_R}{\left(m_g^2-m_5^2\right)^3(m_5-m_R)(m_5+m_R)(m_g-m_R)^3(m_g+m_R)^3}$$

$$\Big(4m_g^2m_R^2\left(m_g^2-m_5^2\right)^3\log(m_R)-4m_5^2m_g^2\log(m_5)\left(m_g^2-m_R^2\right)^3$$

$$+\left(m_5^2-m_R^2\right)\left(4m_g^2\log(m_g)\left(-3m_5^2m_g^2m_R^2+m_5^2m_R^2\left(m_5^2+m_R^2\right)+m_g^6\right)\right.$$

$$\left.+(m_g^2-m_5^2)(m_g^2-m_R^2)\left(m_5^2\left(m_g^2+m_R^2\right)-3m_g^4+m_g^2m_R^2\right)\right)\Big)$$

$$+\frac{5m_5^6-27m_5^4m_R^2+27m_5^2m_R^4-12m_R^4\left(m_R^2-3m_5^2\right)\log\left(\frac{m_5}{m_R}\right)-5m_R^6}{2\left(m_5^2-m_R^2\right)^4}$$

$$+\frac{5\left(-5m_g^6+27m_g^4m_R^2-27m_g^2m_R^4+12\left(m_g^6-3m_g^4m_R^2\right)\log\left(\frac{m_g}{m_R}\right)+5m_R^6\right)}{3\left(m_g^2-m_R^2\right)^4}$$

$$-\frac{5m_g^6-27m_g^4m_R^2+27m_g^2m_R^4+12\left(m_g^6-3m_g^4m_R^2\right)\log\left(\frac{m_R}{m_g}\right)-5m_R^6}{3\left(m_g^2-m_R^2\right)^4}, \tag{C.185}$$

$$c_{42R}=\frac{1}{3}\Bigg(\frac{108m_gm_R}{\left(m_g^2-m_5^2\right)^3(m_5-m_R)^2(m_5+m_R)^2(m_g-m_R)^2(m_g+m_R)^2}$$

$$\Big(-2m_R^4\left(m_5^2-m_g^2\right)^3\log(m_R)$$

$$+\left(m_R^2-m_5^2\right)\left((m_5^2-m_g^2)(m_g^2-m_R^2)\left(2m_5^2m_g^2-m_R^2\left(m_5^2+m_g^2\right)\right)\right.$$

$$\left.+2m_g^2(m_5^2-m_R^2)\log(m_g)\left(m_5^2\left(m_g^2-2m_R^2\right)+m_g^4\right)\right)$$

$$+2m_5^2\log(m_5)\left(m_g^2-m_R^2\right)^2\left(m_5^4+m_5^2m_g^2-2m_g^2m_R^2\right)\Big)$$

$$+\frac{3\left(-2m_5^6+9m_5^4m_R^2-18m_5^2m_R^4+12m_R^6\log\left(\frac{m_5}{m_R}\right)+11m_R^6\right)}{\left(m_5^2-m_R^2\right)^4}$$

$$+\frac{-12m_g^6\log\left(\frac{m_g}{m_R}\right)+11m_g^6-18m_g^4m_R^2+9m_g^2m_R^4-2m_R^6}{\left(m_g^2-m_R^2\right)^4}$$

$$-\frac{7m_g^6-27m_g^2m_R^4+12\left(m_g^6-6m_g^4m_R^2+6m_g^2m_R^4\right)\log\left(\frac{m_g}{m_R}\right)+20m_R^6}{\left(m_g^2-m_R^2\right)^4}$$

$$+\frac{10\left(12m_g^6\log\left(\frac{m_R}{m_g}\right)+11m_g^6-18m_g^4m_R^2+9m_g^2m_R^4-2m_R^6\right)}{\left(m_g^2-m_R^2\right)^4}\Bigg), \tag{C.186}$$

$$
\begin{aligned}
c_{43R} = & \frac{18 m_g m_R}{\left(m_g^2 - m_5^2\right)^3 (m_5 - m_R)^3 (m_5 + m_R)^3 (m_g - m_R)(m_g + m_R)} \\
& \Big(4m_5^2 \Big(m_R^2 \left(m_g^2 - m_5^2\right)^3 \log(m_R) + m_g^2 \left(m_5^2 - m_R^2\right)^3 \log(m_g) \\
& + \log(m_5)(m_R^2 - m_g^2) \left(m_5^6 - 3m_5^2 m_g^2 m_R^2 + m_g^2 m_R^2 \left(m_g^2 + m_R^2\right)\right)\Big) \\
& + (m_5^2 - m_g^2)(m_5^2 - m_R^2)(m_g^2 - m_R^2) \left(3m_5^4 - m_5^2 \left(m_g^2 + m_R^2\right) - m_g^2 m_R^2\right)\Big) \\
& + \frac{5m_5^6 - 27m_5^4 m_R^2 + 27 m_5^2 m_R^4 - 12 m_R^4 \left(m_R^2 - 3m_5^2\right) \log\left(\frac{m_5}{m_R}\right) - 5m_R^6}{2\left(m_5^2 - m_R^2\right)^4} \\
& + \frac{5\left(-5m_g^6 + 27m_g^4 m_R^2 - 27 m_g^2 m_R^4 + 12\left(m_g^6 - 3m_g^4 m_R^2\right) \log\left(\frac{m_g}{m_R}\right) + 5m_R^6\right)}{3\left(m_g^2 - m_R^2\right)^4} \\
& + \frac{-12 m_g^6 \log\left(\frac{m_R}{m_g}\right) - 11 m_g^6 + 18 m_g^4 m_R^2 - 9 m_g^2 m_R^4 + 2m_R^6}{3\left(m_g^2 - m_R^2\right)^4}.
\end{aligned}
\tag{C.187}
$$

C.2.3 Coefficients in $\mathcal{O}_{qqGG}$

The q-q-G-G operator is written as

$$
\begin{aligned}
\mathcal{O}_{qqGG}(x) = \frac{g_s^4}{192\pi^2} \int & \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\frac{d^4k_4}{(2\pi)^4}(2\pi)^4\delta^4(-k_1+k_2+k_3+k_4) \\
& \overline{q}(k_1)\left[F^{\mu\nu}_{L,R}\delta^{ab} + H^{\mu\nu}_{L,R}T^aT^b\right] G^a_\mu(k_2) G^b_\nu(k_3) P_{L,R} q(k_4),
\end{aligned}
\tag{C.188}
$$

where $E_i^\mu,\ F_i^{\mu\nu},\ H_i^{\mu\nu},\ (i = L,\ R)$ are

$$
F_i^{\mu\nu} = f_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + f_{2i\alpha} g^{\mu\nu}\gamma^\alpha + f_{3i\alpha} g^{\alpha\mu}\gamma^\nu + f_{4i\alpha} g^{\alpha\nu}\gamma^\mu, \tag{C.189}
$$

$$
H_i^{\mu\nu} = h_{1i\alpha} i\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta + h_{2i\alpha} g^{\mu\nu}\gamma^\alpha + h_{3i\alpha} g^{\alpha\mu}\gamma^\nu + h_{4i\alpha} g^{\alpha\nu}\gamma^\mu. \tag{C.190}
$$

The coefficients of color singlet part, $F_i^{\mu\nu}$, are given as

$$
\begin{aligned}
f_{1R\alpha} = & \{3[g_1(m_g,m_R) + g_4(m_g,m_R) + g_7(m_g,m_R)] + h_1(m_g,m_R) - h_4(m_g,m_R) - h_7(m_g,m_R) \\
& - \frac{1}{2}[g_1(m_5,m_R) + g_4(m_5,m_R) + g_7(m_5,m_R) - h_1(m_5,m_R) + h_4(m_5,m_R) - h_7(m_5,m_R)] \\
& + \frac{1}{4}[i_1(m_g,m_R) - 2j_1(m_g,m_R,m_5) + 2s_1(m_g,m_R)] \\
& + \frac{1}{2}[s_8(m_g,m_R,m_5) - s_{12}(m_g,m_R,m_5)]\Big\} k_{1\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&+\{3[g_2(m_g,m_R)+g_5(m_g,m_R)+g_8(m_g,m_R)]+h_2(m_g,m_R)-h_5(m_g,m_R)-h_8(m_g,m_R)\\
&-\frac{1}{2}[g_2(m_5,m_R)+g_5(m_5,m_R)+g_8(m_5,m_R)-h_2(m_5,m_R)+h_5(m_5,m_R)-h_8(m_5,m_R)]\\
&+\frac{1}{4}[i_2(m_g,m_R)+2j_2(m_g,m_R,m_5)-2s_1(m_g,m_R)+n_1(m_g,m_R)+2n_{11}(m_g,m_R)]\\
&\left.-\frac{1}{2}[s_8(m_g,m_R,m_5)-s_{12}(m_g,m_R,m_5)]\right\}k_{3\alpha}\\
&+\{3[g_3(m_g,m_R)+g_6(m_g,m_R)+g_9(m_g,m_R)]+h_3(m_g,m_R)-h_6(m_g,m_R)-h_9(m_g,m_R)\\
&-\frac{1}{2}[g_3(m_5,m_R)+g_6(m_5,m_R)+g_9(m_5,m_R)-h_3(m_5,m_R)+h_6(m_5,m_R)-h_9(m_5,m_R)]\\
&+\frac{1}{4}[i_3(m_g,m_R)+2j_3(m_g,m_R,m_5)-2s_1(m_g,m_R)\\
&\left.-\frac{1}{2}[s_8(m_g,m_R,m_5)-s_{12}(m_g,m_R,m_5)]\right\}k_{4\alpha},
\end{aligned}
\tag{C.191}
$$

$$
\begin{aligned}
f_{2R\alpha}=&\{-3g_1(m_g,m_R)+3g_4(m_g,m_R)+2g_7(m_g,m_R)+h_1(m_g,m_R)-h_4(m_g,m_R)-h_7(m_g,m_R)\\
&-\frac{1}{2}[g_1(m_5,m_R)-g_4(m_5,m_R)+g_7(m_5,m_R)-h_1(m_5,m_R)+h_4(m_5,m_R)-h_7(m_5,m_R)]\\
&+\frac{1}{4}[i_4(m_g,m_R)-2j_1(m_g,m_R,m_5)-2j_1(m_5,m_R,m_g)]\\
&-3[e_1(m_g,m_R,m_5)-e_1(m_5,m_R,m_g)]\\
&+\frac{1}{4}[n_2(m_g,m_R)+2n_{12}(m_g,m_R)+l_1(m_R,m_5)-2s_2(m_g,m_R)]\\
&\left.-\frac{1}{2}[r_1(m_g,m_R,m_5)+r_{10}(m_g,m_R,m_5)-s_8(m_g,m_R,m_5)+s_{12}(m_g,m_R,m_5)]\right\}k_{1\alpha}\\
&+\{-3g_2(m_g,m_R)+3g_5(m_g,m_R)+2g_8(m_g,m_R)+h2(m_g,m_R)-h5(m_g,m_R)-h_8(m_g,m_R)\\
&-\frac{1}{2}[g_2(m_5,m_R)-g_5(m_5,m_R)+g_8(m_5,m_R)-h_2(m_5,m_R)+h_5(m_5,m_R)-h_8(m_5,m_R)]\\
&+\frac{1}{4}[i_5(m_5,m_R)-2j_2(m_g,m_R,m_5)-2j_2(m_5,m_R,m_g)\\
&+n_3(m_g,m_R)+2n_{13}(m_g,m_R)+l_2(m_R,m_5)-2s_3(m_g,m_R)]\\
&\left.+\frac{1}{2}[-r_2(m_g,m_R,m_5)-r_{11}(m_g,m_R,m_5)-s_8(m_g,m_R,m_5)+s_{12}(m_g,m_R,m_5)]\right\}k_{3\alpha}\\
&+\{-3g_3(m_g,m_R)+3g_6(m_g,m_R)+2g_9(m_g,m_R)+h3(m_g,m_R)-h6(m_g,m_R)-h_9(m_g,m_R)\\
&-\frac{1}{2}[g_3(m_5,m_R)-g_6(m_5,m_R)+g_9(m_5,m_R)-h_3(m_5,m_R)+h_6(m_5,m_R)-h_9(m_5,m_R)]
\end{aligned}
$$

$$+\frac{1}{4}[i_6(m_5,m_R)-2j_3(m_g,m_R,m_5)-2j_3(m_5,m_R,m_g)$$
$$-3[e_2(m_g,m_R,m_5)-e_2(m_5,m_R,m_g)]$$
$$+\frac{1}{4}[n_4(m_g,m_R)+2n_{14}(m_g,m_R)+l_3(m_R,m_5)-2s_4(m_g,m_R)]$$
$$+\frac{1}{2}[-r_3(m_g,m_R,m_5)-r_{12}(m_g,m_R,m_5)-s_8(m_g,m_R,m_5)+s_{12}(m_g,m_R,m_5)]\Bigg\}\,k_{4\alpha}, \tag{C.192}$$

$$f_{3R\alpha}=\Bigg\{\frac{1}{4}[g_1(m_g,m_R)+4g_4(m_g,m_R)-8g_7(m_g,m_R)+4(h_1(m_g,m_R)+h_4(m_g,m_R)+h_7(m_g,m_R))]$$
$$-\frac{1}{2}[g_1(m_5,m_R)+g_4(m_5,m_R)-g_7(m_5,m_R)-h_1(m_5,m_R)-h_4(m_5,m_R)+h_7(m_5,m_R)]$$
$$+\frac{1}{4}(i_7(m_5,m_R)-2j_1(m_g,m_R,m_5))-3e_1(m_g,m_R,m_5)$$
$$+\frac{1}{4}(n_5(m_g,m_R)+2n_{12}(m_g,m_R))+\frac{1}{4}[l_4(m_R,m_5)-2s_2(m_g,m_R)]$$
$$+\frac{1}{2}[-t_1(m_g,m_R,m_5)-r_4(m_g,m_R,m_5)-r_{13}(m_g,m_R,m_5)$$
$$+s_9(m_g,m_R,m_5)-s_{13}(m_g,m_R,m_5)]\Big\}\,k_{1\alpha}$$
$$+\Bigg\{\frac{1}{4}[g_2(m_g,m_R)+4g_5(m_g,m_R)-8g_8(m_g,m_R)+4(h_2(m_g,m_R)+h_5(m_g,m_R)+h_8(m_g,m_R))]$$
$$-\frac{1}{2}[g_2(m_5,m_R)+g_5(m_5,m_R)-g_8(m_5,m_R)-h_2(m_5,m_R)-h_5(m_5,m_R)+h_8(m_5,m_R)]$$
$$+\frac{1}{4}[i_8(m_5,m_R)+2j_2(m_g,m_R,m_5)+n_6(m_g,m_R)+2n_{13}(m_g,m_R)]$$
$$+\frac{1}{4}l_5(m_R,m_5)+\frac{1}{2}[-s_3(m_g,m_R)-t_2(m_g,m_R,m_5)-r_5(m_g,m_R,m_5)-r_{14}(m_g,m_R,m_5)$$
$$+s_{10}(m_g,m_R,m_5)-s_{14}(m_g,m_R,m_5)]\Big\}\,k_{3\alpha}$$
$$+\Bigg\{\frac{1}{4}[g_3(m_g,m_R)+4g_6(m_g,m_R)-8g_9(m_g,m_R)+4(h_3(m_g,m_R)+h_6(m_g,m_R)+h_9(m_g,m_R))]$$
$$-\frac{1}{2}(g_3(m_5,m_R)+g_6(m_5,m_R)-g_9(m_5,m_R)-h_3(m_5,m_R)-h_6(m_5,m_R)+h_9(m_5,m_R))$$
$$+\frac{1}{4}[i_9(m_5,m_R)+2j_3(m_g,m_R,m_5)]-3e_2(m_5,m_R,m_g)+\frac{1}{4}[n_7(m_g,m_R)+2n_{14}(m_g,m_R)]$$
$$+\frac{1}{4}l_6(m_R,m_5)+\frac{1}{2}[-s_4(m_g,m_R,m_5)-t_3(m_g,m_R,m_5)-r_6(m_g,m_R,m_5)-r_{15}(m_g,m_R,m_5)$$
$$+s_{11}(m_g,m_R,m_5)-s_{15}(m_g,m_R,m_5)]\Big\}\,k_{4\alpha}, \tag{C.193}$$

$$
\begin{aligned}
f_{4R\alpha} = &\left\{\frac{1}{4}[12g_1(m_g,m_R) - 12g_4(m_g,m_R) + g_7(m_g,m_R)\right.\\
&+ 4(-h_1(m_g,m_R) + h_4(m_g,m_R) + h_7(m_g,m_R))]\\
&- \frac{1}{2}(-g_1(m_5,m_R) + g_4(m_5,m_R) + g_7(m_5,m_R) + h_1(m_5,m_R) - h_4(m_5,m_R) - h_7(m_5,m_R))\\
&+ \frac{1}{4}[i_{10}(m_5,m_R) - 2j_1(m_g,m_R,m_5)] - 3e_1(m_5,m_R,m_g) + \frac{1}{4}[n_8(m_g,m_R) + l_7(m_R,m_5)]\\
&+ \frac{1}{2}[s_5(m_g,m_R) + s_2(m_g,m_R)] + \frac{1}{2}[-t_4(m_g,m_R,m_5) - r_7(m_g,m_R,m_5) - r_{16}(m_g,m_R,m_5)\\
&\left. -s_8(m_g,m_R,m_5) + s_{12}(m_g,m_R,m_5)]\right\} k_{1\alpha}\\
&+ \left\{\frac{1}{4}[12g_2(m_g,m_R) - 12g_5(m_g,m_R) + g_8(m_g,m_R)\right.\\
&+ 4(-h_2(m_g,m_R) + h_5(m_g,m_R) + h_8(m_g,m_R))]\\
&- \frac{1}{2}(-g_2(m_5,m_R) + g_5(m_5,m_R) + g_8(m_5,m_R) + h_2(m_5,m_R) - h_5(m_5,m_R) - h_8(m_5,m_R))\\
&+ \frac{1}{4}[i_{11}(m_5,m_R) + 2j_2(m_g,m_R,m_5) + n_9(m_g,m_R) + l_8(m_R,m_5)]\\
&+ \frac{1}{2}[s_6(m_g,m_R) + s_3(m_g,m_R)] + \frac{1}{2}[-t_5(m_g,m_R,m_5) - r_8(m_g,m_R,m_5) - r_{17}(m_g,m_R,m_5)\\
&\left. +s_8(m_g,m_R,m_5) - s_{12}(m_g,m_R,m_5)]\right\} k_{3\alpha}\\
&+ \left\{\frac{1}{4}[12g_3(m_g,m_R) - 12g_6(m_g,m_R) + g_9(m_g,m_R)\right.\\
&+ 4(-h_3(m_g,m_R) + h_6(m_g,m_R) + h_9(m_g,m_R))]\\
&- \frac{1}{2}(-g_3(m_5,m_R) + g_6(m_5,m_R) + g_9(m_5,m_R) + h_3(m_5,m_R) - h_6(m_5,m_R) - h_9(m_5,m_R))\\
&+ \frac{1}{4}[i_{12}(m_5,m_R) + 2j_3(m_g,m_R,m_5)] - 3e_2(m_5,m_R,m_g) + \frac{1}{4}[n_{10}(m_g,m_R) + l_9(m_R,m_5)]\\
&+ \frac{1}{2}[s_7(m_g,m_R) - s_4(m_g,m_R) - t_6(m_g,m_R)] + \frac{1}{2}[-r_9(m_g,m_R,m_5) - r_{18}(m_g,m_R,m_5)\\
&\left. +s_8(m_g,m_R,m_5) - s_{12}(m_g,m_R,m_5)]\right\} k_{4\alpha}, \qquad \text{(C.194)}\\
f_{1L\alpha} = &\{3[g_1(m_g,m_L) + g_4(m_g,m_L) + g_7(m_g,m_L)] + h_1(m_g,m_L) - h_4(m_g,m_L) - h_7(m_g,m_L)\\
&- \frac{1}{2}[g_1(m_5,m_L) + g_4(m_5,m_L) + g_7(m_5,m_L) - h_1(m_5,m_L) + h_4(m_5,m_L) - h_7(m_5,m_L)]\\
&+ \frac{1}{4}[i_1(m_g,m_L) - 2j_1(m_g,m_L,m_5) + 2s_1(m_g,m_L)]\\
&\left. +\frac{1}{2}[-s_8(m_g,m_L,m_5) + s_{12}(m_g,m_L,m_5)]\right\} k_{1\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&+\{3[g_2(m_g,m_L)+g_5(m_g,m_L)+g_8(m_g,m_L)]+h_2(m_g,m_L)-h_5(m_g,m_L)-h_8(m_g,m_L)\\
&-\frac{1}{2}[g_2(m_5,m_L)+g_5(m_5,m_L)+g_8(m_5,m_L)-h_2(m_5,m_L)+h_5(m_5,m_L)-h_8(m_5,m_L)]\\
&+\frac{1}{4}[i_2(m_g,m_L)+2j_2(m_g,m_L,m_5)-2s_1(m_g,m_L)+n_1(m_g,m_L)+2n_{11}(m_g,m_L)]\\
&-\frac{1}{2}[-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\Big\}k_{3\alpha}\\
&+\{3[g_3(m_g,m_L)+g_6(m_g,m_L)+g_9(m_g,m_L)]+h_3(m_g,m_L)-h_6(m_g,m_L)-h_9(m_g,m_L)\\
&-\frac{1}{2}[g_3(m_5,m_L)+g_6(m_5,m_L)+g_9(m_5,m_L)-h_3(m_5,m_L)+h_6(m_5,m_L)-h_9(m_5,m_L)]\\
&+\frac{1}{4}[i_3(m_g,m_L)+2j_3(m_g,m_L,m_5)-2s_1(m_g,m_L)\\
&-\frac{1}{2}[-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\Big\}k_{4\alpha}, \qquad \text{(C.195)}
\end{aligned}
$$

$$
\begin{aligned}
f_{2L\alpha}=&\{-3g_1(m_g,m_L)+3g_4(m_g,m_L)+2g_7(m_g,m_L)+h_1(m_g,m_L)-h_4(m_g,m_L)-h_7(m_g,m_L)\\
&-\frac{1}{2}[g_1(m_5,m_L)-g_4(m_5,m_L)+g_7(m_5,m_L)-h_1(m_5,m_L)+h_4(m_5,m_L)-h_7(m_5,m_L)]\\
&+\frac{1}{4}[i_4(m_g,m_L)-2j_1(m_g,m_L,m_5)-2j_1(m_5,m_L,m_g)]\\
&-3[e_1(m_g,m_L,m_5)-e_1(m_5,m_L,m_g)]\\
&+\frac{1}{4}[n_2(m_g,m_L)+2n_{12}(m_g,m_L)+l_1(m_L,m_5)-2s_2(m_g,m_L)]\\
&+\frac{1}{2}[r_1(m_g,m_L,m_5)+r_{10}(m_g,m_L,m_5)-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\Big\}k_{1\alpha}\\
&+\{-3g_2(m_g,m_L)+3g_5(m_g,m_L)+2g_8(m_g,m_L)+h2(m_g,m_L)-h5(m_g,m_L)-h_8(m_g,m_L)\\
&-\frac{1}{2}[g_2(m_5,m_L)-g_5(m_5,m_L)+g_8(m_5,m_L)-h_2(m_5,m_L)+h_5(m_5,m_L)-h_8(m_5,m_L)]\\
&+\frac{1}{4}[i_5(m_5,m_L)-2j_2(m_g,m_L,m_5)-2j_2(m_5,m_L,m_g)\\
&+n_3(m_g,m_L)+2n_{13}(m_g,m_L)+l_2(m_L,m_5)-2s_3(m_g,m_L)]\\
&-\frac{1}{2}[-r_2(m_g,m_L,m_5)-r_{11}(m_g,m_L,m_5)-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\Big\}k_{3\alpha}\\
&+\{-3g_3(m_g,m_L)+3g_6(m_g,m_L)+2g_9(m_g,m_L)+h3(m_g,m_L)-h6(m_g,m_L)-h_9(m_g,m_L)\\
&-\frac{1}{2}[g_3(m_5,m_L)-g_6(m_5,m_L)+g_9(m_5,m_L)-h_3(m_5,m_L)+h_6(m_5,m_L)-h_9(m_5,m_L)]\\
&+\frac{1}{4}[i_6(m_5,m_L)-2j_3(m_g,m_L,m_5)-2j_3(m_5,m_L,m_g)\\
&-3[e_2(m_g,m_L,m_5)-e_2(m_5,m_L,m_g)]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{4}[n_4(m_g,m_L)+2n_{14}(m_g,m_L)+l_3(m_L,m_5)-2s_4(m_g,m_L)]\\
&\left.-\frac{1}{2}[-r_3(m_g,m_L,m_5)-r_{12}(m_g,m_L,m_5)-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\right\}k_{4\alpha},
\end{aligned}
\tag{C.196}
$$

$$
\begin{aligned}
f_{3L\alpha}=&\left\{\frac{1}{4}[g_1(m_g,m_L)+4g_4(m_g,m_L)-8g_7(m_g,m_L)+4(h_1(m_g,m_L)+h_4(m_g,m_L)+h_7(m_g,m_L))]\right.\\
&-\frac{1}{2}[g_1(m_5,m_L)+g_4(m_5,m_L)-g_7(m_5,m_L)-h_1(m_5,m_L)-h_4(m_5,m_L)+h_7(m_5,m_L)]\\
&+\frac{1}{4}(i_7(m_5,m_L)-2j_1(m_g,m_L,m_5))-3e_1(m_g,m_L,m_5)\\
&+\frac{1}{4}(n_5(m_g,m_L)+2n_{12}(m_g,m_L))+\frac{1}{4}[l_4(m_L,m_5)-2s_2(m_g,m_L)]\\
&-\frac{1}{2}[-t_1(m_g,m_L,m_5)-r_4(m_g,m_L,m_5)-r_{13}(m_g,m_L,m_5)\\
&\left.+s_9(m_g,m_L,m_5)-s_{13}(m_g,m_L,m_5)]\right\}k_{1\alpha}\\
&+\left\{\frac{1}{4}[g_2(m_g,m_L)+4g_5(m_g,m_L)-8g_8(m_g,m_L)+4(h_2(m_g,m_L)+h_5(m_g,m_L)+h_8(m_g,m_L))]\right.\\
&-\frac{1}{2}[g_2(m_5,m_L)+g_5(m_5,m_L)-g_8(m_5,m_L)-h_2(m_5,m_L)-h_5(m_5,m_L)+h_8(m_5,m_L)]\\
&+\frac{1}{4}[i_8(m_5,m_L)+2j_2(m_g,m_L,m_5)+n_6(m_g,m_L)+2n_{13}(m_g,m_L)]\\
&+\frac{1}{4}l_5(m_L,m_5)-\frac{1}{2}[-s_3(m_g,m_L)-t_2(m_g,m_L,m_5)-r_5(m_g,m_L,m_5)-r_{14}(m_g,m_L,m_5)\\
&\left.+s_{10}(m_g,m_L,m_5)-s_{14}(m_g,m_L,m_5)]\right\}k_{3\alpha}\\
&+\left\{\frac{1}{4}[g_3(m_g,m_L)+4g_6(m_g,m_L)-8g_9(m_g,m_L)+4(h_3(m_g,m_L)+h_6(m_g,m_L)+h_9(m_g,m_L))]\right.\\
&-\frac{1}{2}(g_3(m_5,m_L)+g_6(m_5,m_L)-g_9(m_5,m_L)-h_3(m_5,m_L)-h_6(m_5,m_L)+h_9(m_5,m_L))\\
&+\frac{1}{4}[i_9(m_5,m_L)+2j_3(m_g,m_L,m_5)]-3e_2(m_5,m_L,m_g)+\frac{1}{4}[n_7(m_g,m_L)+2n_{14}(m_g,m_L)]\\
&+\frac{1}{4}l_6(m_L,m_5)-\frac{1}{2}[-s_4(m_g,m_L,m_5)-t_3(m_g,m_L,m_5)-r_6(m_g,m_L,m_5)-r_{15}(m_g,m_L,m_5)\\
&\left.+s_{11}(m_g,m_L,m_5)-s_{15}(m_g,m_L,m_5)]\right\}k_{4\alpha}
\end{aligned}
\tag{C.197}
$$

$$
\begin{aligned}
f_{4L\alpha},=&\left\{\frac{1}{4}[12g_1(m_g,m_L)-12g_4(m_g,m_L)+g_7(m_g,m_L)\right.\\
&+4(-h_1(m_g,m_L)+h_4(m_g,m_L)+h_7(m_g,m_L))]\\
&-\frac{1}{2}(-g_1(m_5,m_L)+g_4(m_5,m_L)+g_7(m_5,m_L)+h_1(m_5,m_L)-h_4(m_5,m_L)-h_7(m_5,m_L))\\
&+\frac{1}{4}[i_{10}(m_5,m_L)-2j_1(m_g,m_L,m_5)]-3e_1(m_5,m_L,m_g)+\frac{1}{4}[n_8(m_g,m_L)+l_7(m_L,m_5)]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}[s_5(m_g,m_L)+s_2(m_g,m_L)]-\frac{1}{2}[-t_4(m_g,m_L,m_5)-r_7(m_g,m_L,m_5)-r_{16}(m_g,m_L,m_5)\\
&-s_8(m_g,m_L,m_5)+s_{12}(m_g,m_L,m_5)]\}\,k_{1\alpha}\\
&+\left\{\frac{1}{4}[12g_2(m_g,m_L)-12g_5(m_g,m_L)+g_8(m_g,m_L)\right.\\
&+4(-h_2(m_g,m_L)+h_5(m_g,m_L)+h_8(m_g,m_L))]\\
&-\frac{1}{2}(-g_2(m_5,m_L)+g_5(m_5,m_L)+g_8(m_5,m_L)+h_2(m_5,m_L)-h_5(m_5,m_L)-h_8(m_5,m_L))\\
&+\frac{1}{4}[i_{11}(m_5,m_L)+2j_2(m_g,m_L,m_5)+n_9(m_g,m_L)+l_8(m_L,m_5)]\\
&+\frac{1}{2}[s_6(m_g,m_L)+s_3(m_g,m_L)]-\frac{1}{2}[-t_5(m_g,m_L,m_5)-r_8(m_g,m_L,m_5)-r_{17}(m_g,m_L,m_5)\\
&+s_8(m_g,m_L,m_5)-s_{12}(m_g,m_L,m_5)]\}\,k_{3\alpha}\\
&+\left\{\frac{1}{4}[12g_3(m_g,m_L)-12g_6(m_g,m_L)+g_9(m_g,m_L)\right.\\
&+4(-h_3(m_g,m_L)+h_6(m_g,m_L)+h_9(m_g,m_L))]\\
&-\frac{1}{2}(-g_3(m_5,m_L)+g_6(m_5,m_L)+g_9(m_5,m_L)+h_3(m_5,m_L)-h_6(m_5,m_L)-h_9(m_5,m_L))\\
&+\frac{1}{4}[i_{12}(m_5,m_L)+2j_3(m_g,m_L,m_5)]-3e_2(m_5,m_L,m_g)+\frac{1}{4}[n_{10}(m_g,m_L)+l_9(m_L,m_5)]\\
&+\frac{1}{2}[s_7(m_g,m_L)-s_4(m_g,m_L)]-\frac{1}{2}[-t_6(m_g,m_L)-r_9(m_g,m_L,m_5)-r_{18}(m_g,m_L,m_5)\\
&+s_8(m_g,m_L,m_5)-s_{12}(m_g,m_L,m_5)]\}\,k_{4\alpha}. \qquad\qquad \text{(C.198)}
\end{aligned}
$$

The coefficients of color octet part, $H_i^{\mu\nu}$, are given as

$$
\begin{aligned}
h_{1R\alpha}=&\left\{-2(g_1(m_g,m_R)+g_4(m_g,m_R)+g_7(m_g,m_R))-\frac{2}{3}(h_1(m_g,m_R)-h_4(m_g,m_R)-h_7(m_g,m_R))\right.\\
&+\frac{1}{3}(g_1(m_5,m_R)+g_4(m_5,m_R)+g_7(m_5,m_R)-h_1(m_5,m_R)+h_4(m_5,m_R)-h_7(m_5,m_R))\\
&\left.+\frac{3}{2}[i_1(m_5,m_R)-2j_1(m_g,m_R,m_5)]-18e_1(m_5,m_R,m_g)\right\}k_{1\alpha}\\
&+\left\{-2(g_2(m_g,m_R)+g_5(m_g,m_R)+g_8(m_g,m_R))-\frac{2}{3}(h_2(m_g,m_R)-h_5(m_g,m_R)-h_8(m_g,m_R))\right.\\
&+\frac{1}{3}(g_2(m_5,m_R)+g_5(m_5,m_R)+g_8(m_5,m_R)-h_2(m_5,m_R)+h_5(m_5,m_R)-h_8(m_5,m_R))\\
&\left.+\frac{3}{2}[i_2(m_5,m_R)+2j_2(m_g,m_R,m_5)]\right\}k_{3\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&+\Big\{-2(g_3(m_g,m_R)+g_6(m_g,m_R)+g_9(m_g,m_R))-\frac{2}{3}(h_3(m_g,m_R)-h_6(m_g,m_R)-h_9(m_g,m_R))\\
&+\frac{1}{3}(g_3(m_5,m_R)+g_6(m_5,m_R)+g_9(m_5,m_R)-h_3(m_5,m_R)+h_6(m_5,m_R)-h_9(m_5,m_R))\\
&+\frac{3}{2}[i_3(m_5,m_R)-2j_3(m_g,m_R,m_5)]-18e_2(m_5,m_R,m_g)\Big\}\,k_{4\alpha},
\end{aligned}
\tag{C.199}
$$

$$
\begin{aligned}
h_{2R\alpha}=&-\Big\{2g_1(m_g,m_R)-2g_4(m_g,m_R)-\frac{4}{3}g_7(m_g,m_R)-\frac{2}{3}(h_1(m_g,m_R)-h_4(m_g,m_R)-h_7(m_g,m_R))\\
&+\frac{1}{3}(g_1(m_5,m_R)-g_4(m_5,m_R)+g_7(m_5,m_R)-h_1(m_5,m_R)+h_4(m_5,m_R)-h_7(m_5,m_R))\\
&+\frac{3}{2}[i_4(m_5,m_R)-2j_1(m_g,m_R,m_5)-2j_1(m_5,m_R,m_g)]-18[e_1(m_g,m_R,m_5)-e_1(m_5,m_R,m_g)]\\
&+\frac{3}{2}[l_1(m_R,m_5)-2r_1(m_g,m_R,m_5)-2r_{10}(m_g,m_R,m_5)]\Big\}\,k_{1\alpha}\\
&-\Big\{2g_2(m_g,m_R)-2g_5(m_g,m_R)-\frac{4}{3}g_8(m_g,m_R)-\frac{2}{3}(h_2(m_g,m_R)-h_5(m_g,m_R)-h_8(m_g,m_R))\\
&+\frac{1}{3}(g_2(m_5,m_R)-g_5(m_5,m_R)+g_8(m_5,m_R)-h_2(m_5,m_R)+h_5(m_5,m_R)-h_8(m_5,m_R))\\
&+\frac{3}{2}[i_5(m_5,m_R)-2j_2(m_g,m_R,m_5)-2j_2(m_5,m_R,m_g)]\\
&-3[r_2(m_g,m_R,m_5)+r_{11}(m_g,m_R,m_5)]\}\,k_{3\alpha}\\
&-\Big\{2g_3(m_g,m_R)-2g_6(m_g,m_R)-\frac{4}{3}g_9(m_g,m_R)-\frac{2}{3}(h_2(m_g,m_R)-h_5(m_g,m_R)-h_8(m_g,m_R))\\
&+\frac{1}{3}(g_3(m_5,m_R)-g_6(m_5,m_R)+g_9(m_5,m_R)-h_3(m_5,m_R)+h_6(m_5,m_R)-h_9(m_5,m_R))\\
&+\frac{3}{2}[i_6(m_5,m_R)-2j_3(m_g,m_R,m_5)-2j_3(m_5,m_R,m_g)]-18[e_3(m_g,m_R,m_5)-e_3(m_5,m_R,m_g)]\\
&+\frac{3}{2}[l_3(m_R,m_5)-2r_3(m_g,m_R,m_5)-2r_{12}(m_g,m_R,m_5)]\Big\}\,k_{4\alpha},
\end{aligned}
\tag{C.200}
$$

$$
\begin{aligned}
h_{3R\alpha}=&-\frac{1}{6}(g_1(m_g,m_R)+4g_4(m_g,m_R)-8g_7(m_g,m_R)+4(h_1(m_g,m_R)+h_4(m_g,m_R)+h_7(m_g,m_R)))\\
&+\Big\{\frac{1}{3}g_1(m_5,m_R)+g_4(m_5,m_R)-g_7(m_5,m_R)-h_1(m_5,m_R)-h_4(m_5,m_R)+h_7(m_5,m_R))\\
&+\frac{3}{2}[i_7(m_5,m_R)-2j_1(m_g,m_R,m_5)]-18e_1(m_g,m_R,m_5)+\frac{3}{2}l_4(m_R,m_5)\\
&-3(t_1(m_5,m_R)+r_4(m_g,m_R,m_5)+r_{13}(m_g,m_R,m_5))\}\,k_{1\alpha}\\
&-\frac{1}{6}(g_2(m_g,m_R)+4g_5(m_g,m_R)-8g_8(m_g,m_R)+4(h_2(m_g,m_R)+h_5(m_g,m_R)+h_8(m_g,m_R)))\\
&+\Big\{\frac{1}{3}g_2(m_5,m_R)+g_5(m_5,m_R)-g_8(m_5,m_R)-h_2(m_5,m_R)-h_5(m_5,m_R)+h_8(m_5,m_R))
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{3}{2}[i_8(m_5,m_R)+2j_2(m_g,m_R,m_5)]+\frac{3}{2}l_5(m_R,m_5) \\
&-3(t_2(m_5,m_R)+r_5(m_g,m_R,m_5)+r_{14}(m_g,m_R,m_5))\Big\}\,k_{3\alpha} \\
&-\frac{1}{6}(g_3(m_g,m_R)+4g_6(m_g,m_R)-8g_9(m_g,m_R)+4(h_3(m_g,m_R)+h_6(m_g,m_R)+h_9(m_g,m_R))) \\
&+\Big\{\frac{1}{3}g_3(m_5,m_R)+g_6(m_5,m_R)-g_9(m_5,m_R)-h_4(m_5,m_R)-h_6(m_5,m_R)+h_9(m_5,m_R)) \\
&+\frac{3}{2}[i_9(m_5,m_R)-2j_3(m_g,m_R,m_5)]-18e_3(m_g,m_R,m_5)+\frac{3}{2}l_6(m_R,m_5) \\
&-3(t_3(m_5,m_R)+r_6(m_g,m_R,m_5)+r_{15}(m_g,m_R,m_5))\Big\}\,k_{4\alpha}, \qquad \text{(C.201)} \\
h_{4R\alpha}=&-\Big\{\frac{1}{6}(12g_1(m_g,m_R)-12g_4(m_g,m_R)+g_7(m_g,m_R) \\
&+4(-h_1(m_g,m_R)+h_4(m_g,m_R)+h_7(m_g,m_R))) \\
&+\frac{1}{3}(-g_1(m_5,m_R)+g_4(m_5,m_R)+g_7(m_5,m_R)+h_1(m_5,m_R)-h_4(m_5,m_R)-h_7(m_5,m_R)) \\
&+\frac{3}{2}[i_{10}(m_5,m_R)-2j_1(m_g,m_R,m_5)]-18e_1(m_g,m_R,m_5)+\frac{3}{2}l_7(m_R,m_5) \\
&-3\,(t_4(m_5,m_R)+r_7(m_g,m_R,m_5)+r_{16}(m_g,m_R,m_5))\Big\}\,k_{1\alpha} \\
&-\Big\{\frac{1}{6}(12g_2(m_g,m_R)-12g_5(m_g,m_R)+g_8(m_g,m_R) \\
&+4(-h_2(m_g,m_R)+h_5(m_g,m_R)+h_8(m_g,m_R))) \\
&+\frac{1}{3}(-g_2(m_5,m_R)+g_5(m_5,m_R)+g_8(m_5,m_R)+h_2(m_5,m_R)-h_5(m_5,m_R)-h_8(m_5,m_R)) \\
&+\frac{3}{2}[i_{11}(m_5,m_R)-2j_2(m_g,m_R,m_5)]+\frac{3}{2}l_8(m_R,m_5) \\
&-3\,(t_5(m_5,m_R)+r_8(m_g,m_R,m_5)+r_{17}(m_g,m_R,m_5))\Big\}\,k_{3\alpha} \\
&-\Big\{\frac{1}{6}(12g_3(m_g,m_R)-12g_6(m_g,m_R)+g_9(m_g,m_R) \\
&+4(-h_3(m_g,m_R)+h_6(m_g,m_R)+h_9(m_g,m_R))) \\
&+\frac{1}{3}(-g_3(m_5,m_R)+g_6(m_5,m_R)+g_9(m_5,m_R)+h_3(m_5,m_R)-h_6(m_5,m_R)-h_9(m_5,m_R)) \\
&+\frac{3}{2}[i_{12}(m_5,m_R)-2j_3(m_g,m_R,m_5)]-18e_3(m_g,m_R,m_5)+\frac{3}{2}l_9(m_R,m_5) \\
&-3\,(t_6(m_5,m_R)+r_9(m_g,m_R,m_5)+r_{18}(m_g,m_R,m_5))\Big\}\,k_{4\alpha}, \qquad \text{(C.202)} \\
h_{1L\alpha}=&\,h_{1R\alpha} \qquad \text{(C.203)}
\end{aligned}
$$

$$
\begin{aligned}
h_{2L\alpha} = &-\Big\{2g_1(m_g,m_L) - 2g_4(m_g,m_L) - \frac{4}{3}g_7(m_g,m_L) - \frac{2}{3}(h_1(m_g,m_L) - h_4(m_g,m_L) - h_7(m_g,m_L))\\
&+ \frac{1}{3}(g_1(m_5,m_L) - g_4(m_5,m_L) + g_7(m_5,m_L) - h_1(m_5,m_L) + h_4(m_5,m_L) - h_7(m_5,m_L))\\
&+ \frac{3}{2}[i_4(m_5,m_L) - 2j_1(m_g,m_L,m_5) - 2j_1(m_5,m_L,m_g)] - 18[e_1(m_g,m_L,m_5) - e_1(m_5,m_L,m_g)]\\
&+ \frac{3}{2}[l_1(m_L,m_5) + 2r_1(m_g,m_L,m_5) + 2r_{10}(m_g,m_L,m_5)]\Big\}\, k_{1\alpha}\\
&-\Big\{2g_2(m_g,m_L) - 2g_5(m_g,m_L) - \frac{4}{3}g_8(m_g,m_L) - \frac{2}{3}(h_2(m_g,m_L) - h_5(m_g,m_L) - h_8(m_g,m_L))\\
&+ \frac{1}{3}(g_2(m_5,m_L) - g_5(m_5,m_L) + g_8(m_5,m_L) - h_2(m_5,m_L) + h_5(m_5,m_L) - h_8(m_5,m_L))\\
&+ \frac{3}{2}[i_5(m_5,m_L) - 2j_2(m_g,m_L,m_5) - 2j_2(m_5,m_L,m_g)]\\
&+ 3[r_2(m_g,m_L,m_5) + r_{11}(m_g,m_L,m_5)]\}\, k_{3\alpha}\\
&-\Big\{2g_3(m_g,m_L) - 2g_6(m_g,m_L) - \frac{4}{3}g_9(m_g,m_L) - \frac{2}{3}(h_2(m_g,m_L) - h_5(m_g,m_L) - h_8(m_g,m_L))\\
&+ \frac{1}{3}(g_3(m_5,m_L) - g_6(m_5,m_L) + g_9(m_5,m_L) - h_3(m_5,m_L) + h_6(m_5,m_L) - h_9(m_5,m_L))\\
&+ \frac{3}{2}[i_6(m_5,m_L) - 2j_3(m_g,m_L,m_5) - 2j_3(m_5,m_L,m_g)] - 18[e_3(m_g,m_L,m_5) - e_3(m_5,m_L,m_g)]\\
&+ \frac{3}{2}[l_3(m_L,m_5) + 2r_3(m_g,m_L,m_5) + 2r_{12}(m_g,m_L,m_5)]\Big\}\, k_{4\alpha}, \qquad \text{(C.204)}
\end{aligned}
$$

$$
\begin{aligned}
h_{3L\alpha} = &-\frac{1}{6}(g_1(m_g,m_L) + 4g_4(m_g,m_L) - 8g_7(m_g,m_L) + 4(h_1(m_g,m_L) + h_4(m_g,m_L) + h_7(m_g,m_L)))\\
&+ \Big\{\frac{1}{3}g_1(m_5,m_L) + g_4(m_5,m_L) - g_7(m_5,m_L) - h_1(m_5,m_L) - h_4(m_5,m_L) + h_7(m_5,m_L))\\
&+ \frac{3}{2}[i_7(m_5,m_L) - 2j_1(m_g,m_L,m_5)] - 18e_1(m_g,m_L,m_5) + \frac{3}{2}l_4(m_L,m_5)\\
&+3(t_1(m_5,m_L) + r_4(m_g,m_L,m_5) + r_{13}(m_g,m_L,m_5))\}\, k_{1\alpha}\\
&- \frac{1}{6}(g_2(m_g,m_L) + 4g_5(m_g,m_L) - 8g_8(m_g,m_L) + 4(h_2(m_g,m_L) + h_5(m_g,m_L) + h_8(m_g,m_L)))\\
&+ \Big\{\frac{1}{3}g_2(m_5,m_L) + g_5(m_5,m_L) - g_8(m_5,m_L) - h_2(m_5,m_L) - h_5(m_5,m_L) + h_8(m_5,m_L))\\
&+ \frac{3}{2}[i_8(m_5,m_L) + 2j_2(m_g,m_L,m_5)] + \frac{3}{2}l_5(m_L,m_5)\\
&+3(t_2(m_5,m_L) + r_5(m_g,m_L,m_5) + r_{14}(m_g,m_L,m_5))\}\, k_{3\alpha}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{6}(g_3(m_g, m_L) + 4g_6(m_g, m_L) - 8g_9(m_g, m_L) + 4(h_3(m_g, m_L) + h_6(m_g, m_L) + h_9(m_g, m_L))) \\
&+ \left\{ \frac{1}{3} g_3(m_5, m_L) + g_6(m_5, m_L) - g_9(m_5, m_L) - h_4(m_5, m_L) - h_6(m_5, m_L) + h_9(m_5, m_L)) \right. \\
&+ \frac{3}{2}[i_9(m_5, m_L) - 2j_3(m_g, m_L, m_5)] - 18e_3(m_g, m_L, m_5) + \frac{3}{2} l_6(m_L, m_5) \\
&\left. +3(t_3(m_5, m_L) + r_6(m_g, m_L, m_5) + r_{15}(m_g, m_L, m_5)) \right\} k_{4\alpha}, \qquad \text{(C.205)}
\end{aligned}
$$

$$
\begin{aligned}
h_{4L\alpha} = &- \left\{ \frac{1}{6}(12g_1(m_g, m_L) - 12g_4(m_g, m_L) + g_7(m_g, m_L) \right. \\
&+ 4(-h_1(m_g, m_L) + h_4(m_g, m_L) + h_7(m_g, m_L))) \\
&+ \frac{1}{3}(-g_1(m_5, m_L) + g_4(m_5, m_L) + g_7(m_5, m_L) + h_1(m_5, m_L) - h_4(m_5, m_L) - h_7(m_5, m_L)) \\
&+ \frac{3}{2}[i_{10}(m_5, m_L) - 2j_1(m_g, m_L, m_5)] - 18e_1(m_g, m_L, m_5) + \frac{3}{2} l_7(m_L, m_5) \\
&\left. + 3\, (t_4(m_5, m_L) + r_7(m_g, m_L, m_5) + r_{16}(m_g, m_L, m_5)) \right\} k_{1\alpha} \\
&- \left\{ \frac{1}{6}(12g_2(m_g, m_L) - 12g_5(m_g, m_L) + g_8(m_g, m_L) \right. \\
&+ 4(-h_2(m_g, m_L) + h_5(m_g, m_L) + h_8(m_g, m_L))) \\
&+ \frac{1}{3}(-g_2(m_5, m_L) + g_5(m_5, m_L) + g_8(m_5, m_L) + h_2(m_5, m_L) - h_5(m_5, m_L) - h_8(m_5, m_L)) \\
&+ \frac{3}{2}[i_{11}(m_5, m_L) - 2j_2(m_g, m_L, m_5)] + \frac{3}{2} l_8(m_L, m_5) \\
&\left. + 3\, (t_5(m_5, m_L) + r_8(m_g, m_L, m_5) + r_{17}(m_g, m_L, m_5)) \right\} k_{3\alpha} \\
&- \left\{ \frac{1}{6}(12g_3(m_g, m_L) - 12g_6(m_g, m_L) + g_9(m_g, m_L) \right. \\
&+ 4(-h_3(m_g, m_L) + h_6(m_g, m_L) + h_9(m_g, m_L))) \\
&+ \frac{1}{3}(-g_3(m_5, m_L) + g_6(m_5, m_L) + g_9(m_5, m_L) + h_3(m_5, m_L) - h_6(m_5, m_L) - h_9(m_5, m_L)) \\
&+ \frac{3}{2}[i_{12}(m_5, m_L) - 2j_3(m_g, m_L, m_5)] - 18e_3(m_g, m_L, m_5) + \frac{3}{2} l_9(m_L, m_5) \\
&\left. + 3\, (t_6(m_5, m_L) + r_9(m_g, m_L, m_5) + r_{18}(m_g, m_L, m_5)) \right\} k_{4\alpha}. \qquad \text{(C.206)}
\end{aligned}
$$

These coefficients, $g(m_a, m_b), h(m_a, m_b), \cdots$, are written in terms of Feynman integral as follows;

$$
g_1(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 - 6xy^2z^3}{z\,(m_b^2 - m_a^2) + m_a^2}, \qquad \text{(C.207)}
$$

$$
\begin{aligned}
g_2(m_a, m_b) &= g_8(m_a, m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6y^2z^3 - 6yz^3}{z\,(m_b^2 - m_a^2) + m_a^2}, \qquad \text{(C.208)}
\end{aligned}
$$

$$
g_3(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6xy^2z^3 - 6yz^3}{z\,(m_b^2 - m_a^2) + m_a^2}, \qquad \text{(C.209)}
$$

$$
\begin{aligned}
g_4(m_a,m_b) &= g_7(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{6xy^2z^3}{z\left(m_b^2-m_a^2\right)+m_a^2}, && \text{(C.210)} \\
g_5(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6y^2z^3-6yz^3+6yz^2}{z\left(m_b^2-m_a^2\right)+m_a^2}, && \text{(C.211)} \\
g_6(m_a,m_b) &= g_9(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6y^2z^3-6yz^3+6yz^2}{z\left(m_b^2-m_a^2\right)+m_a^2}, && \text{(C.212)} \\
h_1(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6yz^2-6xy^2z^3\right)}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.213)} \\
h_2(m_a,m_b) &= h_8(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6yz^2-6xy^2z^3\right)}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.214)} \\
h_3(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6xy^2z^3-6yz^3\right)}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.215)} \\
h_4(m_a,m_b) &= h_7(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{6xy^2z^3m_b^2}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.216)} \\
h_5(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6y^2z^3-6yz^3+6yz^2\right)}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.217)} \\
h_6(m_a,m_b) &= h_9(m_a,m_b) && \text{(C.218)} \\
&= \frac{m_b^2\left(6xy^2z^3-6yz^3+6yz^2\right)}{\left(z\left(m_b^2-m_a^2\right)+m_a^2\right)^2}, && \text{(C.219)} \\
i_1(m_a,m_b) &= i_4(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{36xy^2z^3+18yz^2}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.220)} \\
i_2(m_a,m_b) &= i_5(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{36\left(yz^3-y^2z^3\right)-48yz^2}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.221)} \\
i_3(m_a,m_b) &= i_6(m_a,m_b) \\
&= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{36\left(yz^3-xy^2z^3\right)-24yz^2}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.222)} \\
i_7(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2-72xy^2z^3}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.223)} \\
i_8(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{72\left(y^2z^3-yz^3\right)+12yz^2}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.224)} \\
i_9(m_a,m_b) &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{72\left(xy^2z^3-yz^3\right)}{z\left(m_a^2-m_b^2\right)+m_b^2}, && \text{(C.225)}
\end{aligned}
$$

$$i_{10}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{-156xy^2z^3 - 18yz^3}{z\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.226}$$

$$i_{11}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{156\left(y^2z^3 - yz^3\right) + 84yz^2}{z\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.227}$$

$$i_{12}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{156\left(xy^2z^3 - yz^3\right) + 132yz^2}{z\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.228}$$

$$j_1(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6xy^2z^3m_b^2}{\left(z\left(m_c^2 - m_a^2\right) + m_a^2 + yz\left(m_b^2 - m_c^2\right)\right)^2}, \tag{C.229}$$

$$j_2(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6yz^3 - 6y^2z^3\right)}{\left(z\left(m_c^2 - m_a^2\right) + m_a^2 + yz\left(m_b^2 - m_c^2\right)\right)^2}, \tag{C.230}$$

$$j_3(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{m_b^2\left(6yz^3 - 6xy^2z^3\right)}{\left(z\left(m_c^2 - m_a^2\right) + m_a^2 + yz\left(m_b^2 - m_c^2\right)\right)^2}, \tag{C.231}$$

$$e_1(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{2xy^2}{y\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.232}$$

$$e_2(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{2y^2 - 2xy^2}{y\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.233}$$

$$n_1(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{12yz^2}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.234}$$

$$\begin{aligned} n_2(m_a, m_b) &= n_8(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6y(4-4z)z^2}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \end{aligned} \tag{C.235}$$

$$n_3(m_a, m_b) = n_9(m_a, m_b) = 0, \tag{C.236}$$

$$\begin{aligned} n_4(m_a, m_b) &= n_{10}(m_a, m_b) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24yz^2(z - yz)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \end{aligned} \tag{C.237}$$

$$n_5(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(8z+4)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.238}$$

$$n_6(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{48yz^2(-xyz + yz - z) - 12yz^2}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.239}$$

$$n_7(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{48yz^2(z - yz)}{yz\left(m_a^2 - m_b^2\right) + m_b^2}, \tag{C.240}$$

$$n_{11}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{6yz^2m_b^2}{\left(yz\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.241}$$

$$n_{12}(m_a, m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12y(1-z)z^2m_b^2}{\left(yz\left(m_a^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.242}$$

$$n_{13}(m_a, m_b) = 0, \tag{C.243}$$

$$n_{14}(m_a,m_b) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2m_b^2(z-yz)}{(yz\,(m_a^2-m_b^2)+m_b^2)^{\,2}}, \tag{C.244}$$

$$t_1(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_am_b(1-2xyz)}{(xyz\,(m_c^2-m_a^2)+yz\,(m_a^2-m_c^2)+z\,(m_c^2-m_b^2)+m_b^2)^{\,2}}, \tag{C.245}$$

$$t_2(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_am_b(1-2xyz)}{(xyz\,(m_c^2-m_a^2)+yz\,(m_a^2-m_c^2)+z\,(m_c^2-m_b^2)+m_b^2)^{\,2}}, \tag{C.246}$$

$$t_3(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_am_b(2(xyz-z)+1)}{(xyz\,(m_c^2-m_a^2)+yz\,(m_a^2-m_c^2)+z\,(m_c^2-m_b^2)+m_b^2)^{\,2}}, \tag{C.247}$$

$$t_4(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12xy^2z^3m_am_b}{(z\,(m_a^2-m_b^2)+xyz\,(m_a^2-m_c^2)+yz\,(m_c^2-m_a^2)+m_b^2)^{\,2}}, \tag{C.248}$$

$$t_5(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2m_am_b(2(yz-z)+1)}{(z\,(m_a^2-m_b^2)+xyz\,(m_a^2-m_c^2)+yz\,(m_c^2-m_a^2)+m_b^2)^{\,2}}, \tag{C.249}$$

$$t_6(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2m_am_b(xyz-z+1)}{(z\,(m_a^2-m_b^2)+xyz\,(m_a^2-m_c^2)+yz\,(m_c^2-m_a^2)+m_b^2)^{\,2}}, \tag{C.250}$$

$$\begin{aligned} l_1(m_b,m_c) &= l_7(m_b,m_c) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{24xy^2z^3}{z\,(m_c^2-m_b^2)+m_b^2}, \end{aligned} \tag{C.251}$$

$$l_2(m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(4z-4yz)}{z\,(m_c^2-m_b^2)+m_b^2}, \tag{C.252}$$

$$l_3(m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(4z-4xyz)}{z\,(m_c^2-m_b^2)+m_b^2}, \tag{C.253}$$

$$l_4(m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(4xyz-2)}{z\,(m_c^2-m_b^2)+m_b^2}, \tag{C.254}$$

$$\begin{aligned} l_5(m_b,m_c) &= l_8(m_b,m_c) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4yz+4z-2)}{z\,(m_c^2-m_b^2)+m_b^2}, \end{aligned} \tag{C.255}$$

$$l_6(m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4xyz+4z-2)}{z\,(m_c^2-m_b^2)+m_b^2}, \tag{C.256}$$

$$l_9(m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4xyz+4z-4)}{z\,(m_c^2-m_b^2)+m_b^2}, \tag{C.257}$$

$$\begin{aligned} r_1(m_a,m_b,m_c) &= r_4(m_a,m_b,m_c) \\ &= \int_0^1 dx \int_0^1 dy \int_0^1 dz - \frac{6xy^2z^3m_am_b}{(z\,(m_a^2-m_b^2)+xyz\,(m_c^2-m_a^2)+m_b^2)^{\,2}}, \end{aligned} \tag{C.258}$$

$$r_2(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (yz-z+1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.259}$$

$$r_3(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (xyz-z+1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.260}$$

$$r_5(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (yz-z)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.261}$$

$$r_6(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (xyz-z+1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.262}$$

$$r_7(m_a,m_b,m_c) = -2r_1(m_a,m_b,m_c), \tag{C.263}$$

$$r_8(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (2yz-2z-1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.264}$$

$$r_9(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{12yz^2 m_a m_b (xyz-z-1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.265}$$

$$r_{10}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6xy^2z^3 m_a m_b}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.266}$$

$$r_{11}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (yz-z-1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.267}$$

$$r_{12}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (xyz-z-1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.268}$$

$$r_{13}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (1-2xyz)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.269}$$

$$r_{14}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (2yz-2z+1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.270}$$

$$r_{15}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (2xyz-2z+1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.271}$$

$$r_{16}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (xyz-1)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.272}$$

$$r_{17}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (yz-z)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.273}$$

$$r_{18}(m_a,m_b,m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b (xyz-z)}{\left(z\left(m_a^2-m_b^2\right)+xyz\left(m_a^2-m_c^2\right)+yz\left(m_c^2-m_a^2\right)+m_b^2\right)^2}, \tag{C.274}$$

$$s_1(m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2}{yz\left(m_c^2 - m_b^2\right) + m_b^2}, \tag{C.275}$$

$$s_2(m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(-4yz + 2z + 1)}{yz\left(m_c^2 - m_b^2\right) + m_b^2}, \tag{C.276}$$

$$s_3(m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(2xyz + 2yz - 2z - 1)}{yz\left(m_c^2 - m_b^2\right) + m_b^2}, \tag{C.277}$$

$$s_4(m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2(2yz - 2z - 1)}{yz\left(m_c^2 - m_b^2\right) + m_b^2}, \tag{C.278}$$

$$\begin{aligned} s_5(m_b, m_c) = &\int_0^1 dx \int_0^1 dy \int_0^1 dz 6yz^2(-4yz + 2z + 2) \\ &\cdot \left(\frac{m_b^2}{\left(yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2} + \frac{1}{yz\left(m_c^2 - m_b^2\right) + m_b^2} \right), \end{aligned} \tag{C.279}$$

$$\begin{aligned} s_6(m_b, m_c) = &\int_0^1 dx \int_0^1 dy \int_0^1 dz 6yz^2(2xyz + 2yz - 2z - 1) \\ &\cdot \left(\frac{m_b^2}{\left(yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2} + \frac{1}{yz\left(m_c^2 - m_b^2\right) + m_b^2} \right), \end{aligned} \tag{C.280}$$

$$\begin{aligned} s_7(m_b, m_c) = &\int_0^1 dx \int_0^1 dy \int_0^1 dz 12yz^2(yz - z) \\ &\cdot \left(\frac{m_b^2}{\left(yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2} + \frac{1}{yz\left(m_c^2 - m_b^2\right) + m_b^2} \right), \end{aligned} \tag{C.281}$$

$$s_8(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b}{\left(xyz\left(m_a^2 - m_c^2\right) + yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.282}$$

$$s_9(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^3 m_a m_b}{\left(xyz\left(m_a^2 - m_c^2\right) + yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.283}$$

$$s_{10}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b(-2xyz + 2yz + 1)}{\left(xyz\left(m_a^2 - m_c^2\right) + yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.284}$$

$$s_{11}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b(2yz - 2z + 1)}{\left(xyz\left(m_a^2 - m_c^2\right) + yz\left(m_c^2 - m_b^2\right) + m_b^2\right)^2}, \tag{C.285}$$

$$s_{12}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b}{\left(yz\left(m_a^2 - m_b^2\right) + xyz\left(m_c^2 - m_a^2\right) + m_b^2\right)^2}, \tag{C.286}$$

$$s_{13}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^3 m_a m_b}{\left(yz\left(m_a^2 - m_b^2\right) + xyz\left(m_c^2 - m_a^2\right) + m_b^2\right)^2}, \tag{C.287}$$

$$s_{14}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b(-2xyz + 2yz + 1)}{\left(yz\left(m_a^2 - m_b^2\right) + xyz\left(m_c^2 - m_a^2\right) + m_b^2\right)^2}, \tag{C.288}$$

$$s_{15}(m_a, m_b, m_c) = \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{6yz^2 m_a m_b(2yz - 2z + 1)}{\left(yz\left(m_a^2 - m_b^2\right) + xyz\left(m_c^2 - m_a^2\right) + m_b^2\right)^2}. \tag{C.289}$$

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